Figure of Merit and Limit of Short-Term Stability in Passive Hydrogen Maser

V.I. Vasilyev, S.A. Kozlov
Institute of Electronic Measurements KVARZ, Nizhny Novgorod, Russia
Email: nnipi_kvarz@sinn.ru

Abstract — **Frequency stability of passive frequency standard is associated with the figure of merit of a quantum discriminator. The method of the discriminator figure of merit measuring and the lowest achievable short-term frequency stability in the passive hydrogen maser are presented. Results of theoretical and experimental research are compared. The result of investigation shows that with increase in a excitation signal power, spectral line shape can be considered as the sum of several lines.**

Keywords - passive hydrogen maser; figure of merit; stability; spectral line shape.

I. INTRODUCTION

Frequency stability of any passive frequency standard is associated with the figure of merit of a quantum discriminator (physics package), characterized by the ratio of signal/noise to the width of the atomic transition spectral line during indicating the atomic resonance. The larger this parameter, the higher is short-term frequency stability of the maser [1]:

$$\sigma(\tau) = \frac{\Delta f_l}{F_{\text{s-n}}} \times \frac{const}{f_l\sqrt{\tau}} \quad ,$$

where $F_{s\text{-}n}$ – signal-to-noise ratio (SNR), Δf_l – width of the atomic spectral line, f_l – frequency of the atomic transition, *const* – coefficient dependent on line detection method, τ – averaging time.

II. THEORETICAL ESTIMATION OF A PASSIVE HYDROGEN MASER STABILITY

Let us consider a passive hydrogen maser (PHM) with single frequency modulation [2]. Block diagram of PHM is given in Fig.1. Expression can be concretized for the quantum discriminator:

$$\sigma(\tau) = \sqrt{\frac{S_n(\Omega)}{2 f_l^2 S_d^2 \tau}},$$

where S_n (Ω) – noise spectral density at the output of the selective amplifier; S_d – the slope of discriminator curve at frequency offset excitation signal from spectral line centre by Δf.

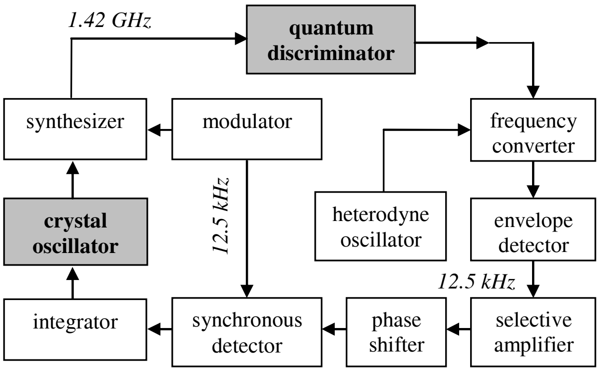

Fig. 1. Block diagram of Passive Hydrogen Maser.

If a selective voltmeter with a pass-band ΔB_{sel}, is used, S_d can be measured as the effective voltage U_{sel} of a first harmonic signal at the output of the selective amplifier, which appears as a result of detuning. U_{n} can be measured as effective noise voltage U_{n} when there is no signal at input. Frequency stability of passive frequency standard can be estimated from the following expression:

$$\sigma(\tau) = \frac{U_n \Delta f}{2\pi \ f_l U_{sel}} \frac{1}{\sqrt{\Delta B_{sel}\tau}} \quad .$$

Experimental dependence of PHM CH1-76A stability at time of averaging 100 sec upon the SNR during frequency offset Δf = 0.1 Hz is given in Fig. 2.

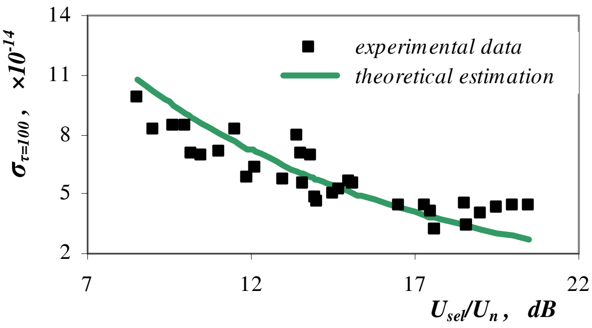

Fig. 2. Dependence of CH1-76A stability at time of averaging 100 sec upon the SNR during frequency offset Δf = 0.1 Hz.

The SNR was measured by means of the selective microvoltmeter with $\Delta B_{sel} = 300$ Hz. Also estimated value calculated by means of the given formula is shown in figure. The analysis of the figure plot shows close agreement of a theoretical estimation and experimental results up to signal/noise values equal to 19 dB, and then there comes limitation on stability improvement.

III. INVESTIGATION OF STABILITY LIMITATION

For investigation of stability limitation we will refer to the slope of discriminator curve S_d. The expression received by us is based on the solution of the equation of PHM [3, 4] in a "steady-state" approximation:

$$S_{\partial} \approx \frac{2J_1(m)}{J_0(m)\sqrt{1+q}}\,\frac{\alpha S_0}{T_1 T_2 (1+S_0)}\,, \qquad (1)$$

where $q = \Omega/\gamma_c$; γ_c – the halfwidth band of the microwave cavity of the hydrogen discriminator; m, Ω – the index and the angular frequency of the phase modulation; $J_n(m)$ – the Bessel function of the first kind of the n order from the argument m; α и S_0 – the hydrogen discriminator excitation and saturation factor; T_1 и T_2 – longitudinal and transverse relaxation time of the atomic ensemble. The expression shows (Fig. 3) that at small saturation the slope of discriminator curve is proportional to S_0 (therefore, to an excitation signal power), and at $S_0 >> 1$ the slope of discriminator curve tends to a limit. But the results of the experiment given in Fig. 4 show that after achievement of a maximum reduction S_d begins at increase of a excitation signal power P_{ex}.

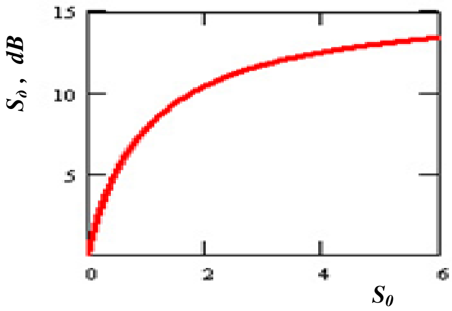

Fig. 3. Theoretical dependence of the slope of discriminator curve upon the hydrogen discriminator saturation factor.

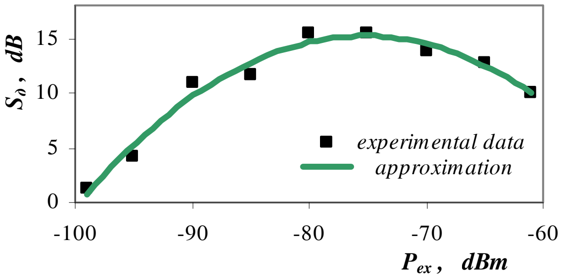

Fig. 4. Experimental dependence of the slope of discriminator curve upon the power of a excitation signal power during frequency offset Δf = 0.1 Hz.

Amplitude characteristics of the quantum hydrogen discriminator were experimentally investigated for verification of theoretical model. According to the conventional representations the atom spectral line has a Lorentz line shape. For example, such result is obtained due to the solution of the equation of a hydrogen maser in a "steady-state" approximation [3].

For verification the equipment with accumulation of data was used. It allowed to increase the accuracy of measurements. As a source of a signal the Anritsu MG3642A signal generator was used. As the receiver the spectrum analyzer E4402B-COM of Agilent Technologies was used. The generator and the spectrum analyzer were synchronized by a highly stable signal of 10 MHz from the active hydrogen standard of frequency CH1-75A. Between the generator and the quantum discriminator the ferrite isolator with the return losses more than 100 dB was placed, and between the discriminator and the receiver the variable attenuator was placed for improvement of wave impedance matching and reduction of signal distortion [5]. We increased attenuation at increase in a signal of the generator.

Experimental data in a form of the hydrogen line shape are given in Fig. 5. Parameter is the power of a excitation signal. The result of investigation shows that the spectral line shape corresponds to Lorentz line shape only at small ($\approx -90 \ldots -100$ dBm) excitation signal power. With increase in a excitation signal power it is observed not only broadening, but also strong distortion of the spectral line. The line shape can be considered as the sum of several (possibly three) spectral lines detuning from each other.

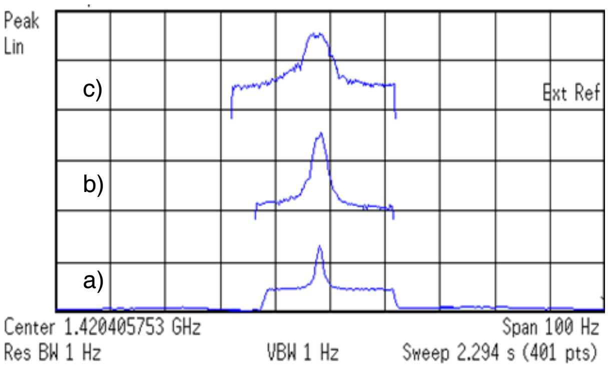

Fig. 5. The PHM spectral line shape at a different excitation signal power P_{ex}: a) – 94 dBm; b) – 74 dBm; c) – 67 dBm.

IV. DISCUSSION

In a "steady-state" approximation it is considered that all atoms give quantum of energy and after that leave the microwave cavity. Actually the quantity of the atoms which have given quantum of energy is defined by induced atom transition rate. The transition rate depends on the power of a excitation signal power. Quantity of atoms which can give and again receive energy quantum grows at increase of signal power.

In these conditions Stark – Zeeman dynamic effect has to be apparent. It is the effect of splitting of the spectral line that appears in strong alternating electromagnetic fields [6, 7]. Splitting of satellites is equally to the Rabi frequency:

$$b_R = \mu_{12}\mu_0 H/\hbar$$

where H – intensity of magnetic microwave field component, μ_{12} – magnetic dipole moment of transition, μ_0 –magnetic constant, $\hbar$ – Planck constant. According to our theoretical estimates at an excitation signal power equal to – 70 dBm we have $b_R/2\pi \approx 2$ Hz.

At a low excitation signal power the Rabi frequency is small and shift of satellites from the central peak is also small in comparison with line width. In this case satellites in Fig. 5 are not observed. Then summation of several lines leads to a flat top of the line. At further increase in Rabi frequency there is an observed splitting of the line.

The analysis shows that there are two main factors defining behavior of the slope of discriminator curve in Fig. 4. The first factor leads to increase in S_d. According to the expression 1 at the small saturation factor S_0 the slope of discriminator curve is proportional to excitation signal power. The second factor is associated with broadening and line splitting, it contributes to S_d reduction. The broadening and splitting leads at the beginning to emergence of a plateau on a discrimination curve near zero frequency offset. After that the general reduction of a tilt angle of the discrimination curve occurs.

V. CONCLUSION

- The larger figure of merit of a quantum discriminator, the higher is short-term frequency stability of the maser.
- At small ($\approx -80 \ldots -100$ dBm) excitation signal power the experimental slope of discriminator curve increases. At further increase of a excitation signal power S_d begins to decrease. The experimental spectral line shape can be considered as the sum of several spectral lines. We explain the spectral line splitting by Stark – Zeeman dynamic effect that appears in strong alternating electromagnetic fields.
- The discovered effect limits short-term stability of passive hydrogen maser.

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