Direct Digital Measurement of Precision Oscillators

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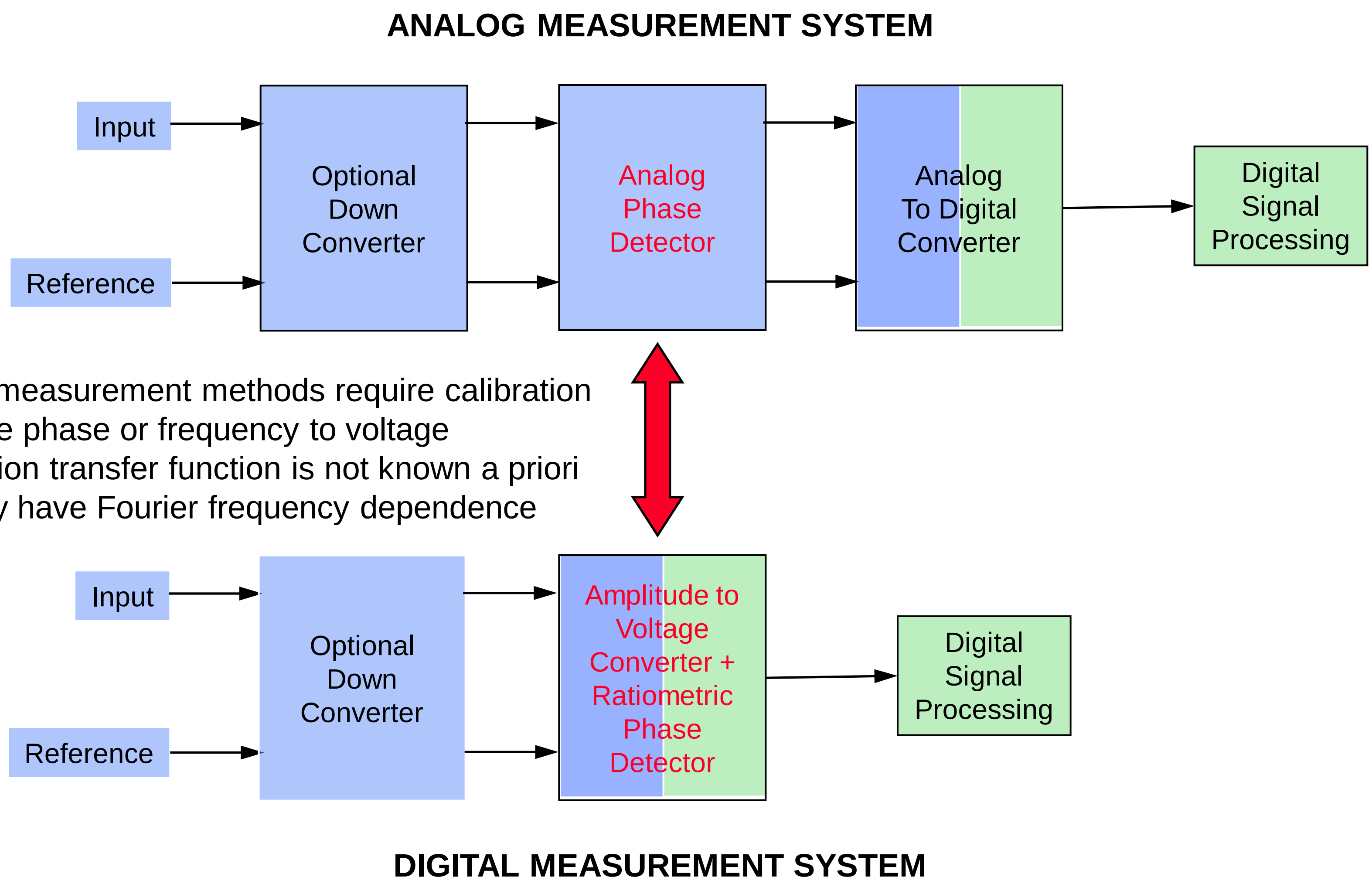
The Essential Difference between Analog and Digital Phase Measurement Systems

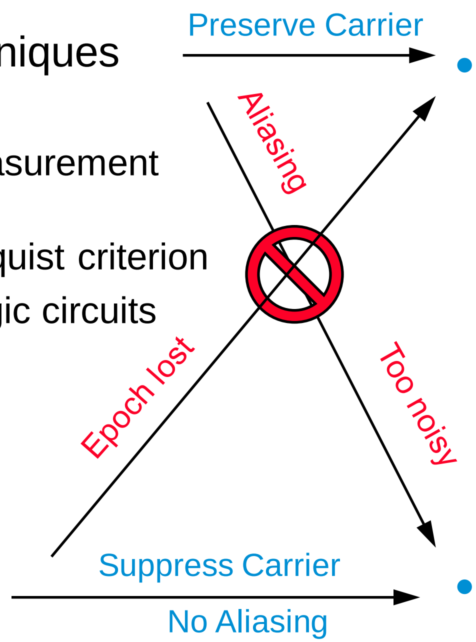


Analog measurement methods require calibration since the phase or frequency to voltage conversion transfer function is not known a priori and may have Fourier frequency dependence

Time and Frequency Domains
Situation through 2004

Measurements

- Counter based techniques
  - Direct counting
  - Heterodyne measurement systems
  - Can't satisfy Nyquist criterion
  - Large jitter in logic circuits
- Analog techniques
  - Mixer phase detectors
  - Delay line discriminators



Analysis

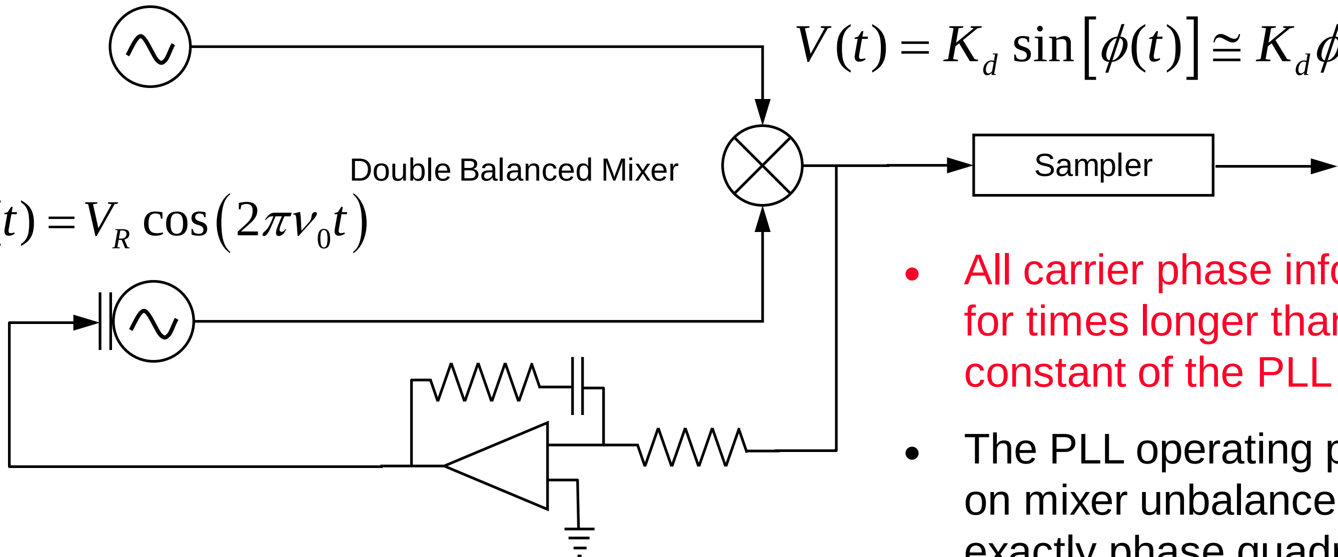
- Time domain
  - Representation of a signal as a time series
  - Statistics that describe the behavior of a signal over time
  - Very long term
- Frequency domain
  - Representation of a signal as a set of spectral components
  - Very low noise
  - Statistics that describe the spectral content of signals

Analog Phase Detector

$$V(t) = [V_0 + \varepsilon(t)]\sin[2\pi\nu_0 t + \phi(t)]$$

$$V(t) = K_d \sin[\phi(t)] \cong K_d \phi(t)$$

$$V(t) = V_R \cos(2\pi\nu_0 t)$$

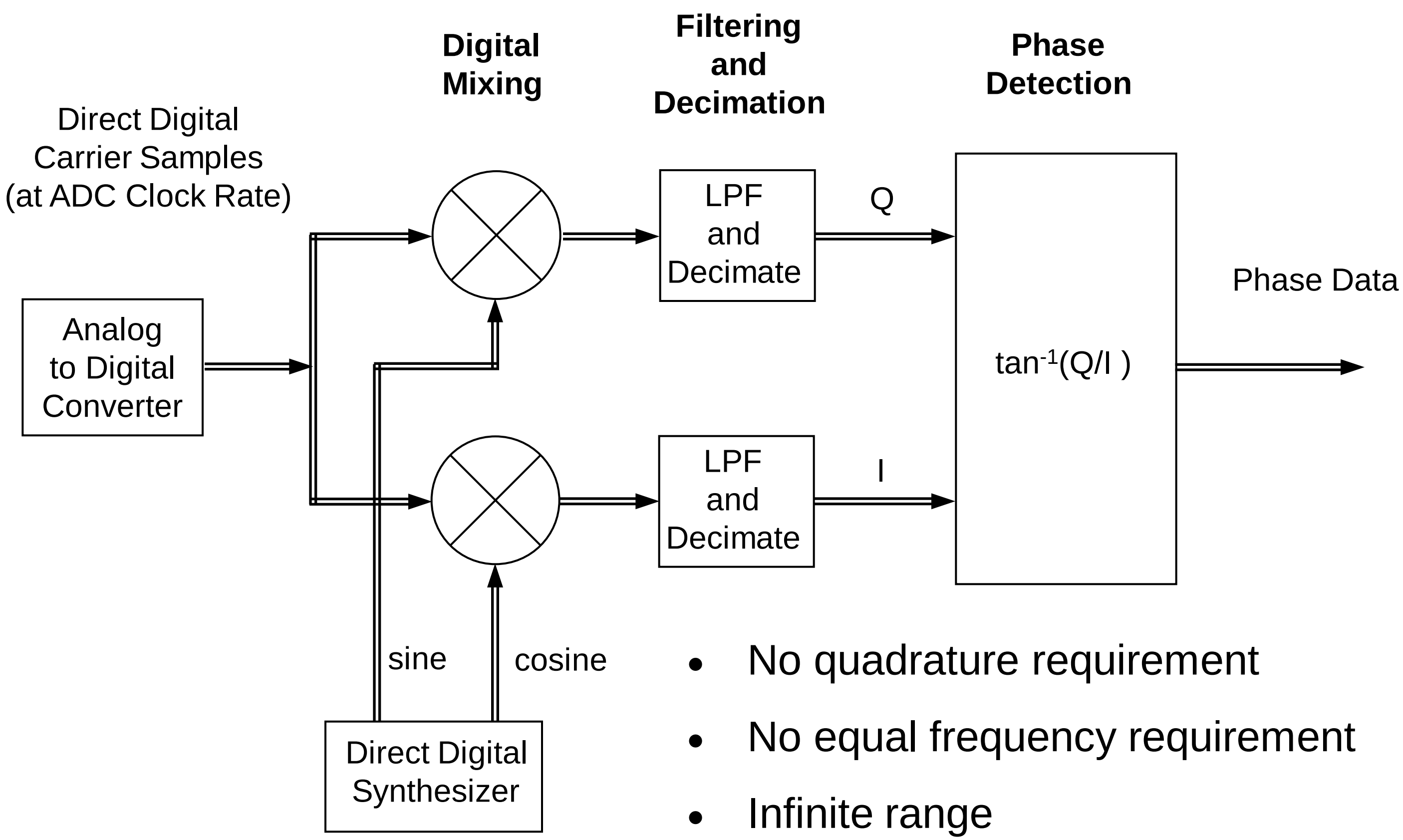


$$\phi_k = \frac{V(t_k)}{K_d}$$

- All carrier phase information is lost for times longer than the time constant of the PLL
- The PLL operating point depends on mixer unbalance and is not exactly phase quadrature so some AM noise is demodulated
- The voltage to phase transfer function must be calibrated for
  - Effect of the PLL
  - Frequency response of the mixer and amplifiers
    - Input impedance
    - Input level

- The Phase Lock Loop maintains the quadrature condition necessary for the double balanced mixer to act as a phase detector
- Carrier suppression makes it possible to measure phase noise 180 dB below the carrier amplitude

Down-Conversion & Digital Phase Detection



- No quadrature requirement
- No equal frequency requirement
- Infinite range
- No user calibration required

Digital Measurement Issues

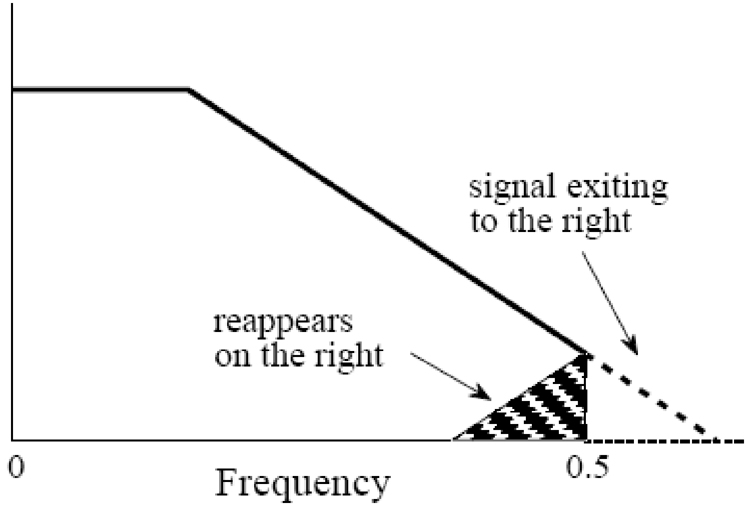
- A non-linear action is taken in the time domain – changing the continuous signal to discrete samples
- Digital information is
  - — Sampled (converts independent variable - time - from continuous to discrete)
  - — Quantized (converts dependent variable from continuous to discrete)
    - » A measurement is not quantized when the noise exceeds the digital resolution
- An analog signal whose frequency is above one-half the sampling rate is
  - — Aliased (appears at a different frequency inside the range
    - » $0 < f < 0.5 \times$ sampling frequency

Quantization Errors

- Each sample can have a maximum error of ± ½ LSB
- The error can look like random noise when the signal value varies multiple bits
  - Mean = 0
  - Standard deviation = LSB/ $\sqrt{12}$
- When the signal value is nearly constant the quantization error remains static for long periods of time
  - Averaging fails
  - Mean error is not zero
  - Dithering the signal results in more ACCURATE average values

Frequency Domain Aliasing

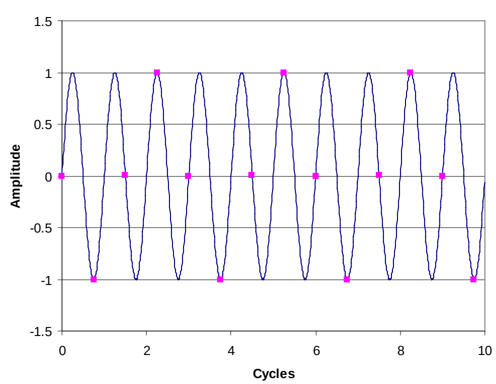
- Sampling theorem (Shannon & Nyquist)
  - A continuous signal can be properly sampled only if it does not contain frequency components above one-half the sampling rate
- When an analog signal is properly sampled, it can be exactly reconstructed from the digital signal
- When a continuous signal's frequency is above one-half the sampling rate, aliasing changes the frequency to something that can be represented in the sampled data



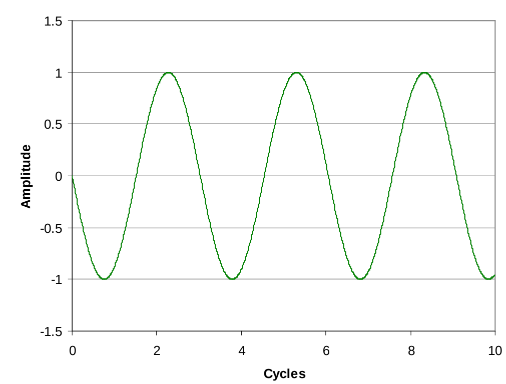
Why Don’t We Make Spectrum Estimates with Heterodyne Measurement Systems? - Aliasing

Conflict

- Aliasing always occurs when the times of zero crossings of a heterodyne signal are used to calculate the phase difference of the RF signals
- Minimum required analog bandwidth > IF frequency (max sampling frequency)
  - — to pass the IF frequency with minimum phase shift
  - — Because we only sample at one polarity zero crossing
- But the sampling theorem requires that
  - — Maximum analog bandwidth < ½ × sampling (IF) frequency



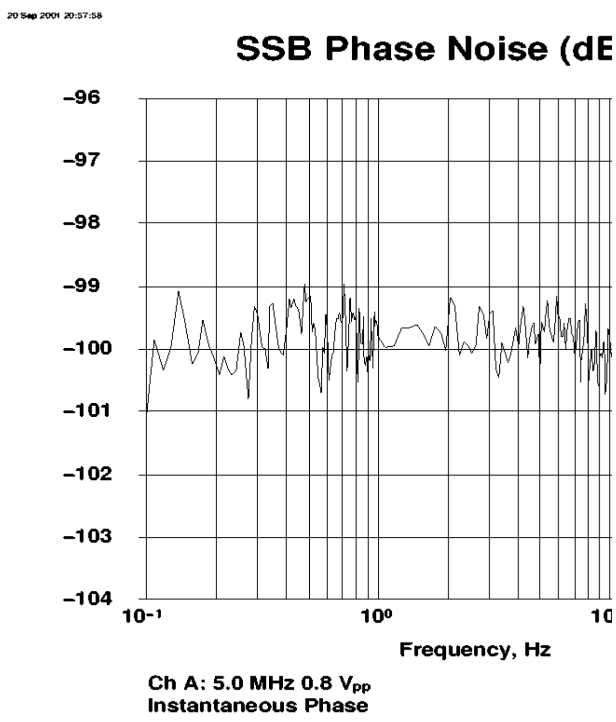
4 samples every 3 cycles



Recovered waveform

Effect of Aliasing on the White Phase Noise Spectrum

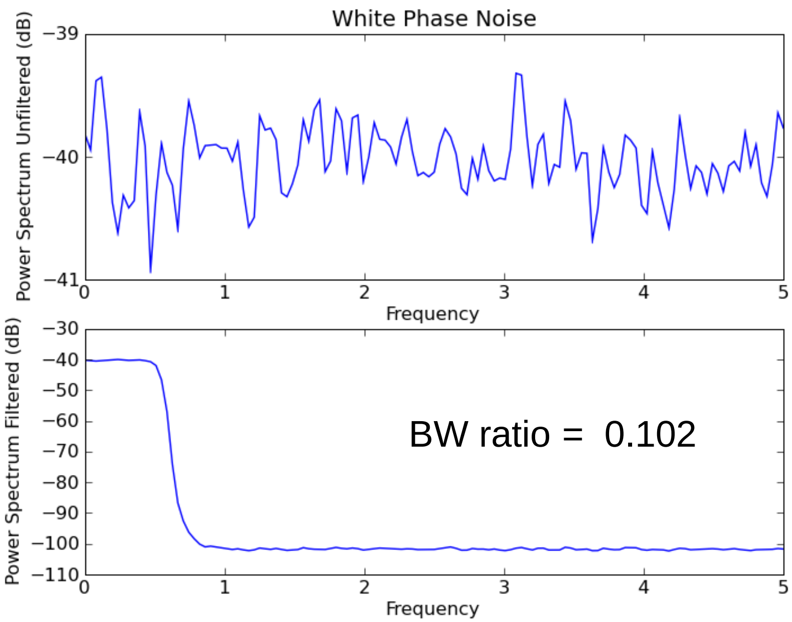
- Sample rate = ν_{IF}
- Bandwidth > $3 \times \nu_{IF}$ in order to pass the 3rd harmonic
- An input of calibrated white phase noise at –110 dBc/Hz results in a measured level of –100 dBc/Hz due to Aliasing

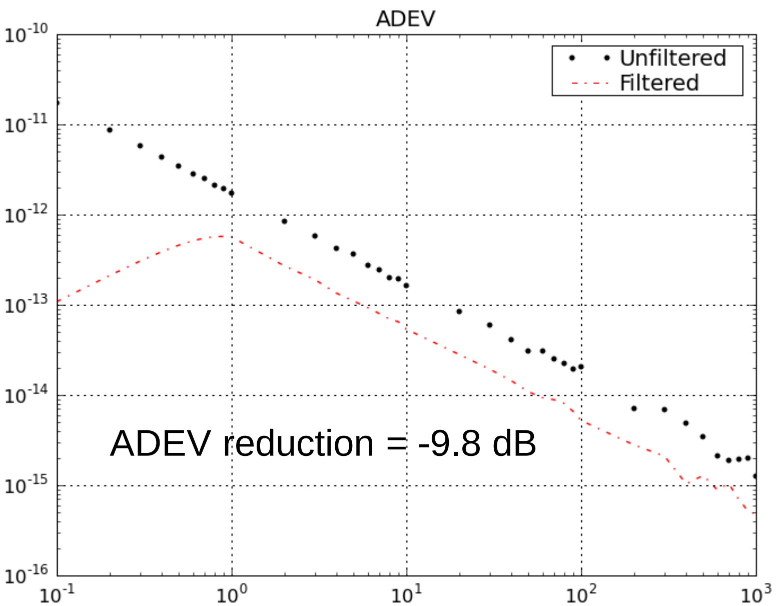


10 dB Aliasing Discrepancy

Effect of Bandwidth on ADEV

- The maximum bandwidth without aliasing is ½(1/sample rate)
- The minimum bandwidth that does not bias the Allan deviation low at τ = sample rate is ½(1/sample rate)
- Therefore, the optimum bandwidth to use for characterizing an oscillator is ½(1/sample rate)



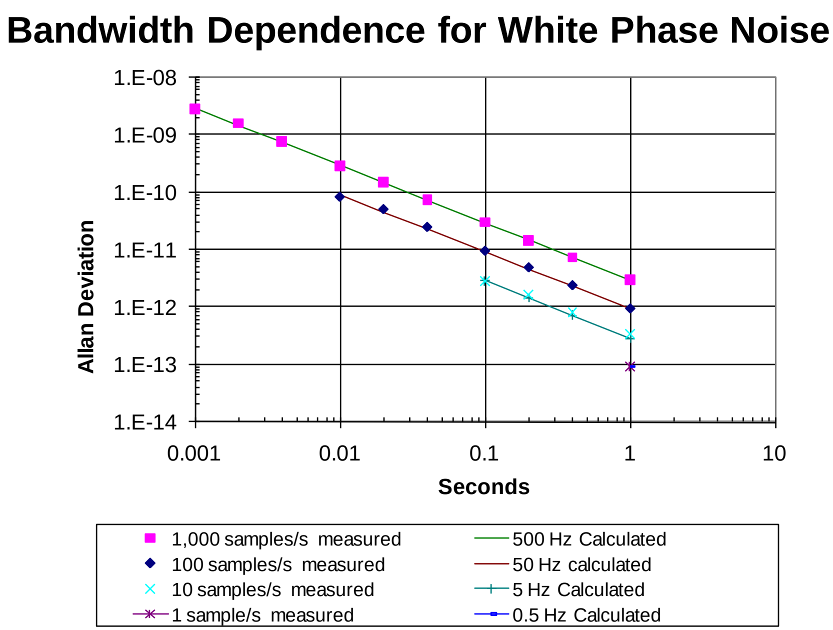


Measured AVAR Bandwidth Dependence

- The Allan deviation may be calculated from the spectral density of phase

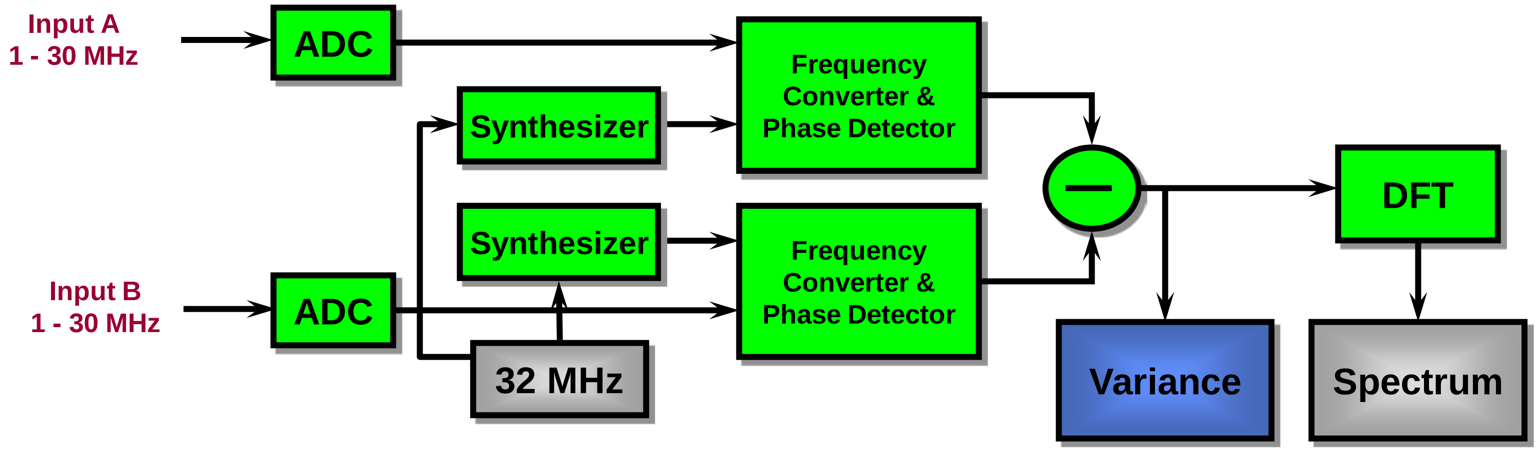
$$\sigma_y(\tau) = \left(2\int_0^{f_h} S_\phi(f)\frac{\sin^4(\pi f \tau)}{(\pi \nu_0 \tau)^2} df \right)^{1/2}$$

- Thus, it depends on the bandwidth, f_h, of the noise in the processor or measurement system



Phase Measurement by RF Sampling (Model 5115A)

Signal Processing Overview for Simultaneous Allan Deviation and Phase Noise

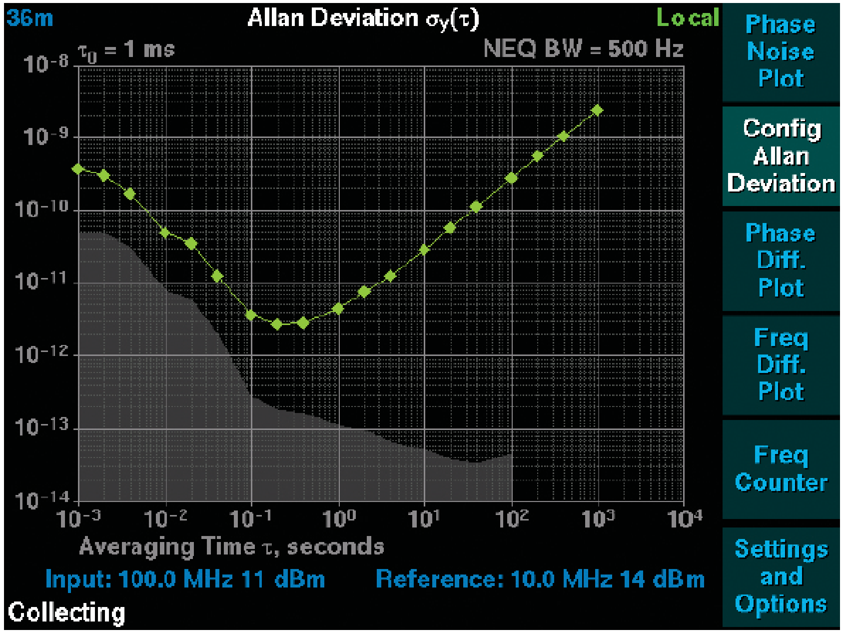


- Uses the dual mixer architecture
  - The phase of the common clock is eliminated by differencing the phase measurements of the two input channels
  - The noises of the ADC’s are added

ADEV of 100-MHz Source *vs.* 10-MHz Ref

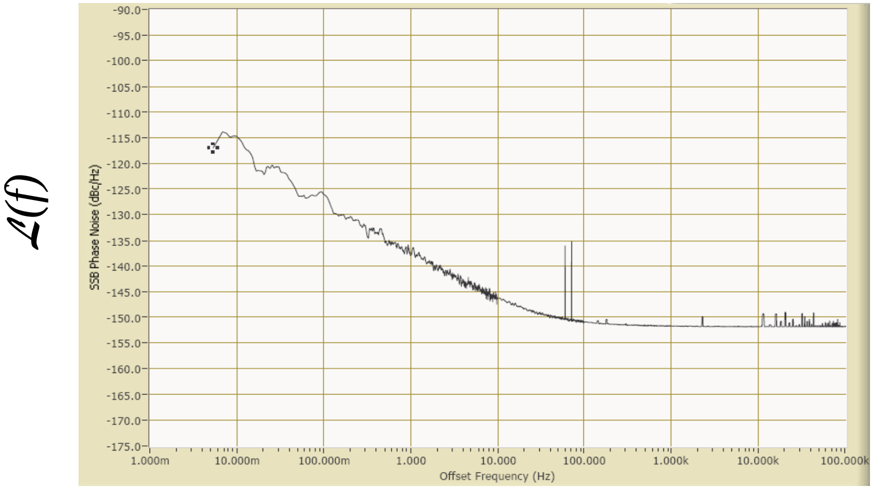
- The Device Under Test and the Reference may have different frequencies
  - The phase measured at input B must be normalized to the frequency of input A

$$\phi_{B\ norm} = \phi_{B\ raw} \frac{\omega_A}{\omega_B}$$

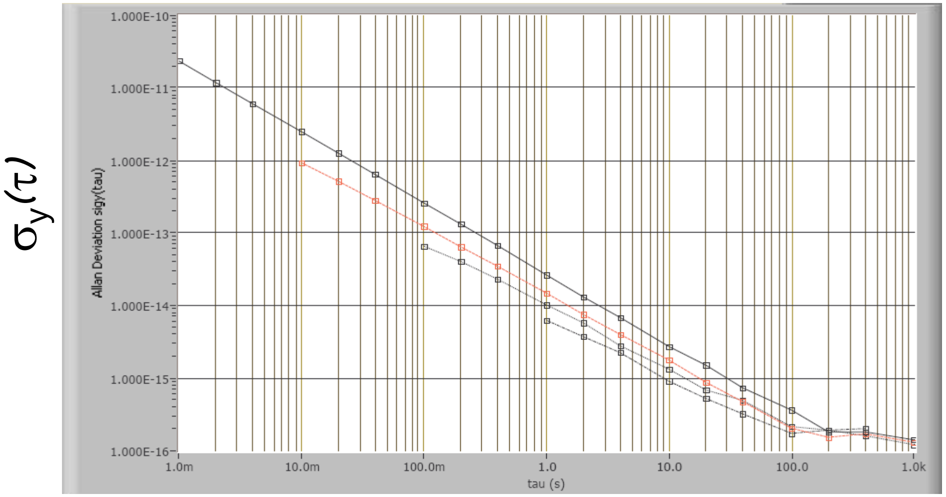


Single Channel Direct Digital Noise Floor at 10 MHz (Model 5115A)

Noise Floor



Variation with Bandwidth

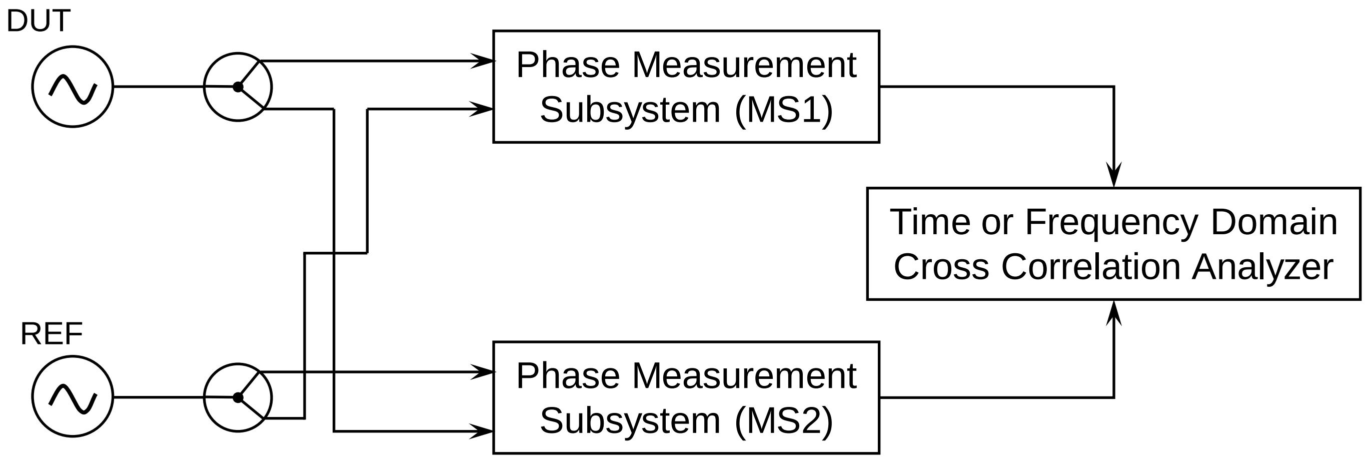


The measurement is the sum of the noise on the DUT and the reference. The noise of either device is guaranteed to be less than the measurement. If the DUT and reference are presumed equal, then the noise of either is the measured noise less 3 dB

The noise bandwidths for the four plots are 500 Hz, 50 Hz, 5 Hz, and 0.5 Hz

Improve the Noise floor with Cross Correlation

- Two measurement systems may be operated in parallel with their outputs going into an analyzer that computes the cross variance or the cross spectrum
- The uncorrelated measurement noise in the two systems approaches an expectation value of zero, improving the noise floor of the measurements
- Improvement goes as the square-root of the number of measurements and is usually limited to 20 – 40 dB by the presence of correlated noise



Cross Correlation Enables Measurement Below the ADC Noise

- Cross AVAR $\sigma_y^{2\otimes}(\tau) \equiv \left\langle \frac{1}{2}\left(\overline{y^1}_{k+1}(\tau) - \overline{y^1}_k(\tau)\right)\left(\overline{y^2}_{k+1}(\tau) - \overline{y^2}_k(\tau)\right)\right\rangle$
- The cross statistics are computed from independent time series of the phase
- The cross AVAR is real but not always positive
- Cross Spectrum
- Although the spectral density is real, the cross spectral density is complex

$$S_\phi^{\otimes}(n\Delta f) = \left\langle \Phi_1(n\Delta f)\Phi_2^*(n\Delta f)\right\rangle$$

- There are alternate candidate methods for estimating the spectrum

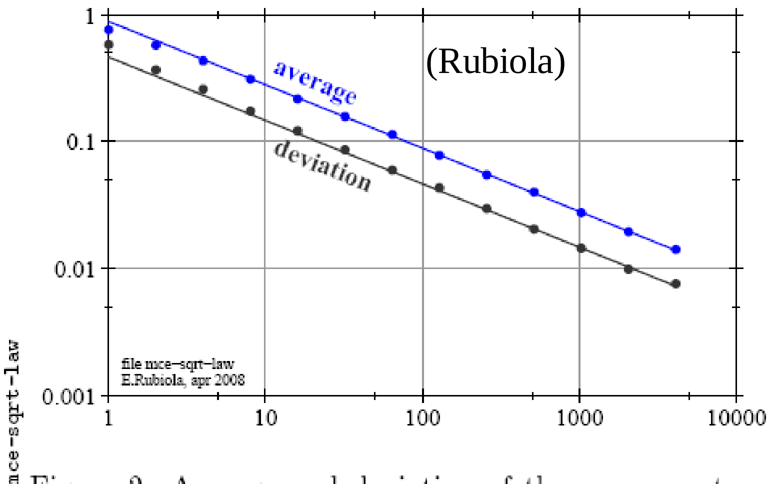


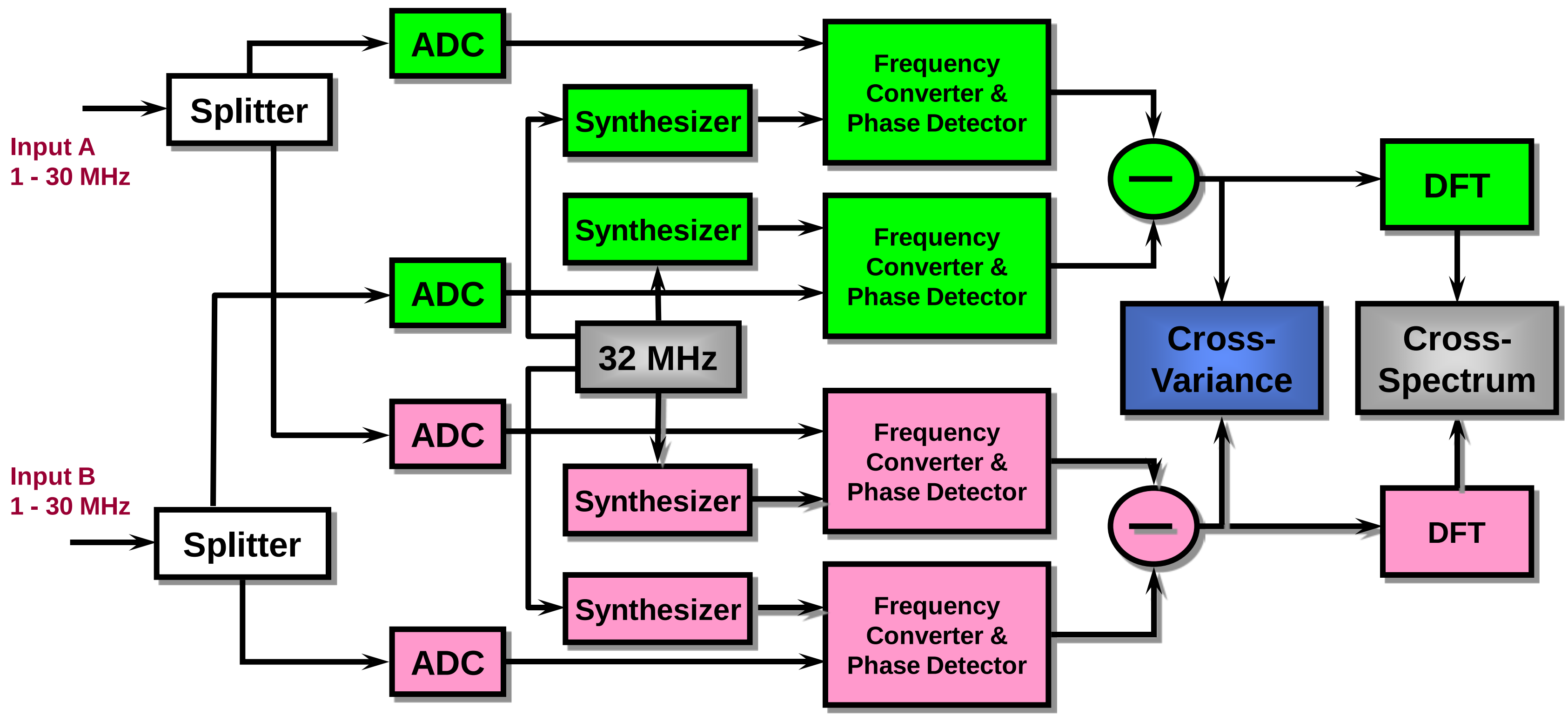
Figure 2: Average and deviation of the cross spectrum $|\langle S_{yx}\rangle_m|$, as s function of the number of averages. The two processes are statistically independent, white, and gaussian distributed. Solid line: theoretical law; dots: simulated data.

$$S_\phi(n\Delta f) \approx \mathrm{Re}\left[S_\phi^{\otimes}(n\Delta f)\right] \qquad \text{unbiased, used for ADEV estimation}$$

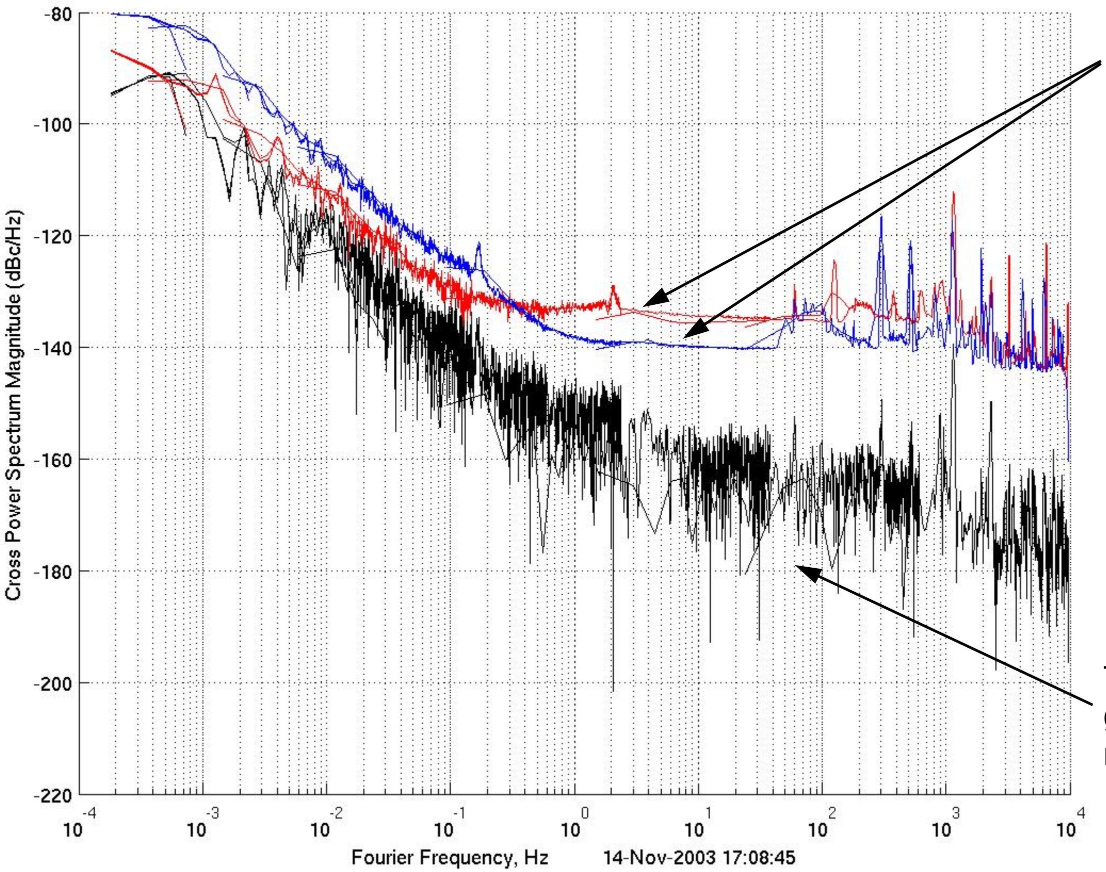
$$S_\phi(n\Delta f) \approx \left|\mathrm{Re}\left[S_\phi^{\otimes}(n\Delta f)\right]\right| \qquad \text{biased, but plots on log-log scale}$$

Digital Phase Measurement System with Noise Reduction by Cross Correlation

Model 5120A



Digital Measurement System Noise Floor
With and Without Cross Correlation

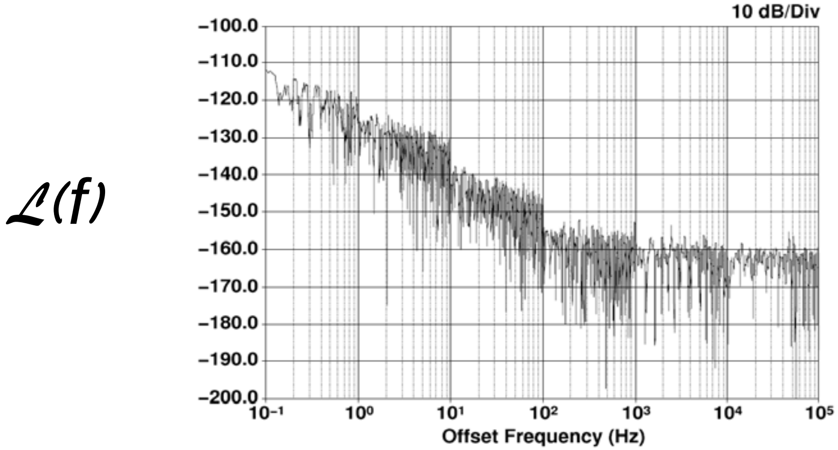


The direct spectrum of the two channels shows evidence of uncorrelated noise

The cross correlation spectrum gives the level of the correlated noise

How Much Cross Correlation is Enough?

- ADEV and spectrum both have uncertainty that varies approximately as $1/\sqrt{N}$
- The instrumentation noise decreases by 5 dB for every factor of 10 increase in the number of measurements

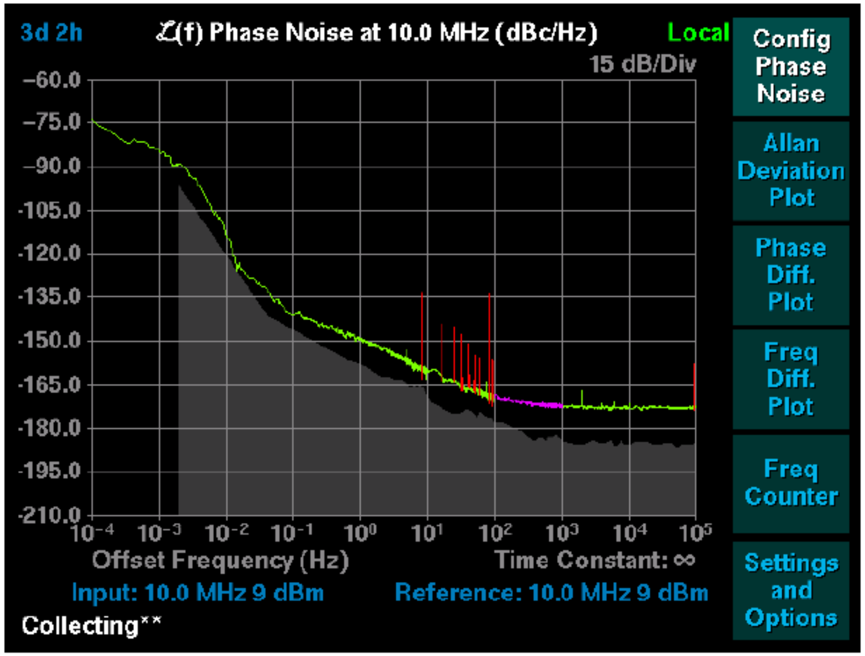


  — The reduction is limited by some phenomenon that is usually described as “correlated” noise but does not have the appearance of correlated noise

- How can one tell if the source noise has been resolved and with what accuracy?

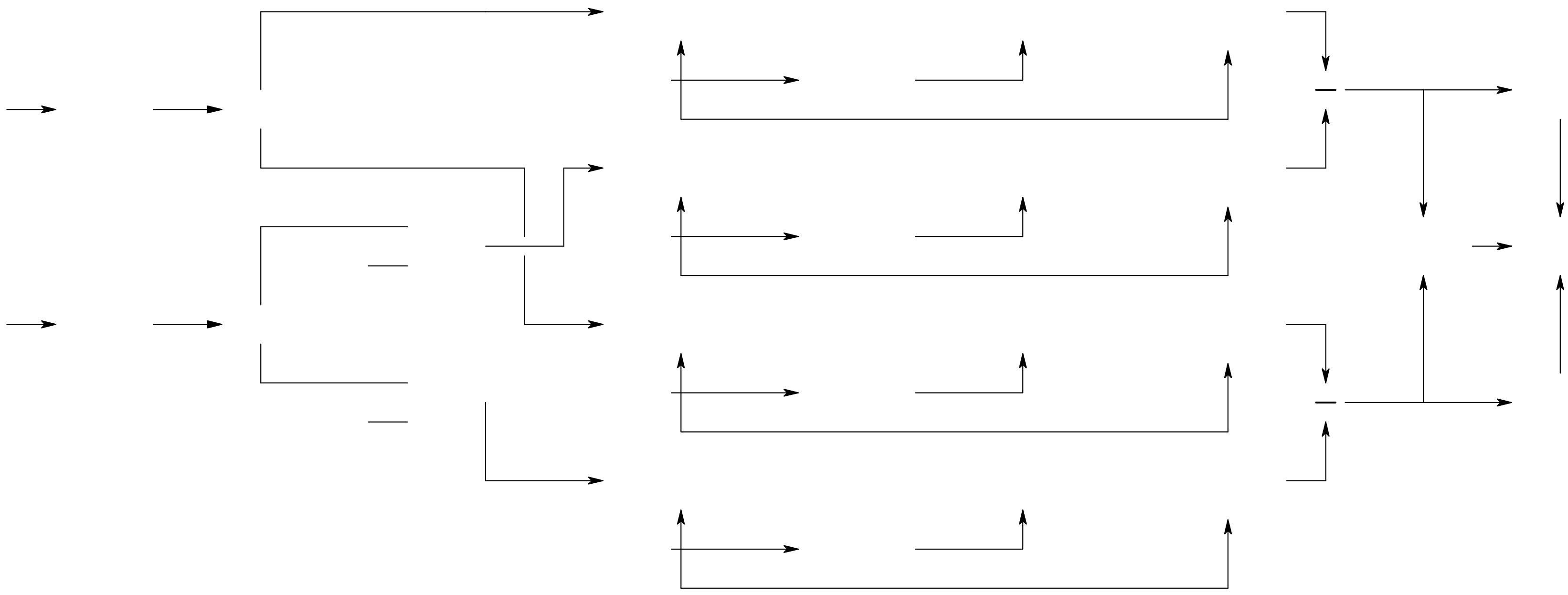
Self Estimation of Noise Floor

- The cross Spectrum is real in the absence of instrument noise
- The imaginary component of the cross spectrum is a good measure of the instrument induced noise
- The integral of the imaginary component is a good estimate of the ADEV noise floor
- 6 dB margin equals
  1 dB bias



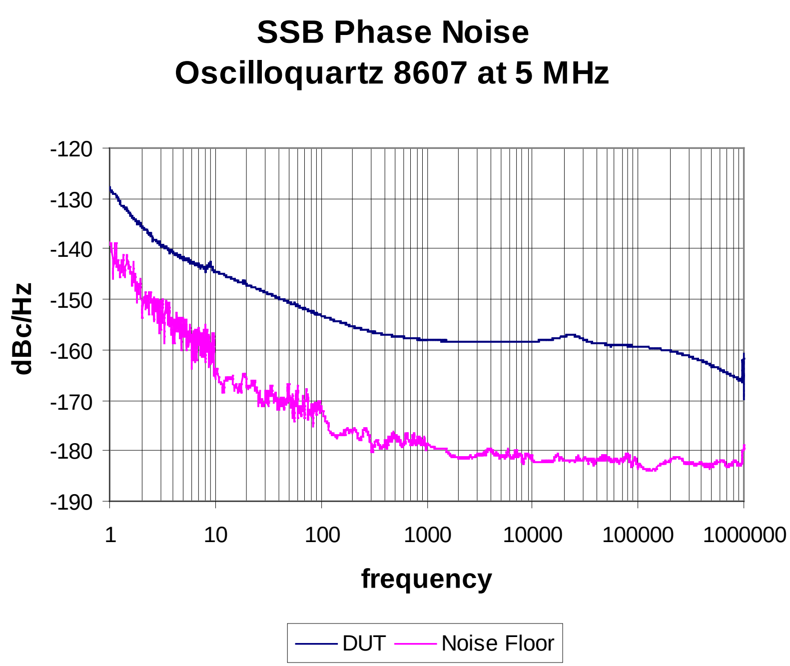
Cross Correlation with Internal Reference Oscillators

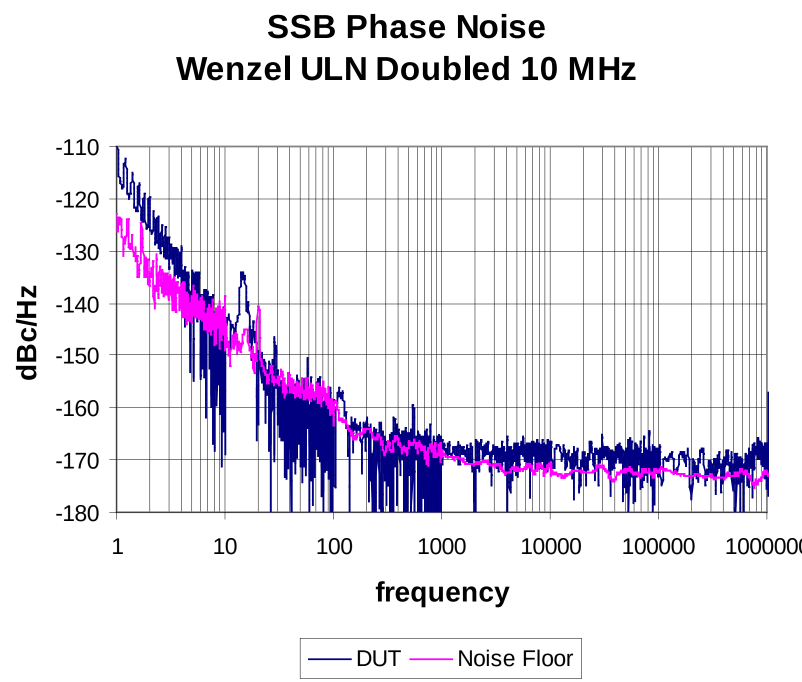
Model 5120A-01



Noise of the internal references is suppressed (separated) by the cross correlation process

Direct Digital Noise Floor with Internal References
(Model 5120A-01 Measuring Model 4145B and 4415A)





- Able to measure
  - — -128 dBc/Hz at 1 Hz offset
  - — -170 dBc/Hz at 1 kHz offset and higher

Cross ADEV with One or Two Reference Oscillators

- The Cross ADEV is an unbiased estimator of the ADEV when an input and reference are applied externally

$$\sigma_y^{2\otimes}(\tau) = \tfrac{1}{2}\left(D_{IN} - D_{REF}\right)^2 \tau^2$$

- The cross ADEV provides a separation of variances when the two internal references are used (1 port measurement)
  - The cross AVAR is an unbiased estimator of the AVAR when source noise dominates over uncorrelated internal reference oscillator noise
  - The cross AVAR is a biased estimator of the AVAR when oscillator aging dominates

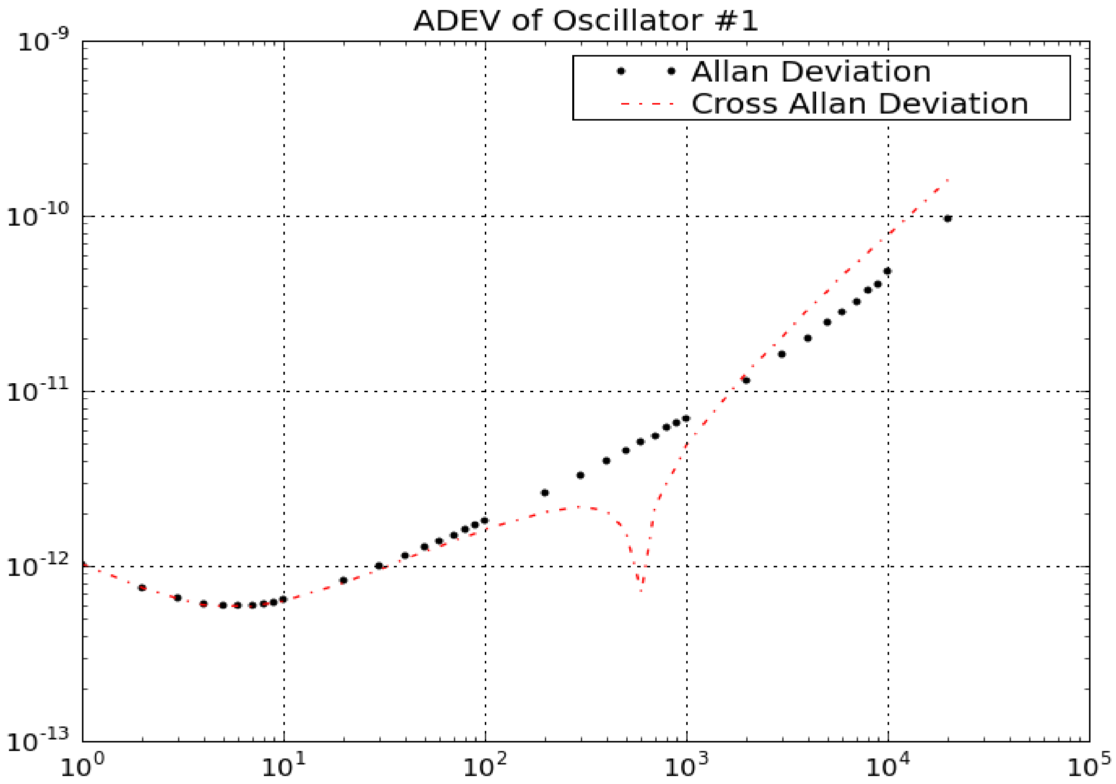
$$\sigma_y^{2\otimes}(\tau) = \tfrac{1}{2}\left(D_{IN} - D_{REF1}\right)\left(D_{IN} - D_{REF2}\right)\tau^2$$

aging of the internal references

  - The cross ADEV can be taken as the square root of the absolute value of the cross AVAR would have a zero at the τ value where the cross AVAR becomes negative

Cross ADEV Differs from ADEV

The Cross ADEV is a biased estimator of the ADEV when the oscillator frequency variation is dominated by aging

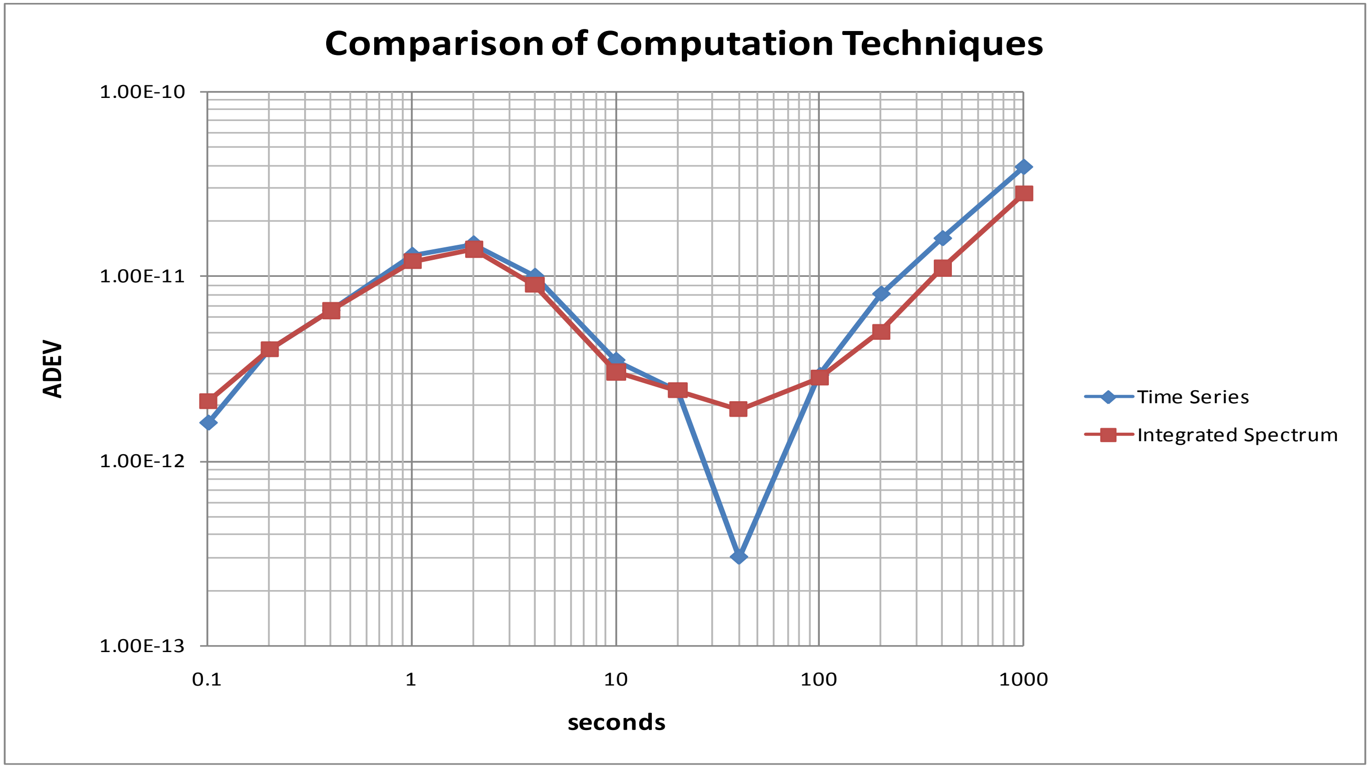


Why does this happen?

Spectrum Fixup of Cross ADEV

- The Allan deviation may be calculated from the cross spectral density of phase to fix this problem

$$\sigma_y(\tau) = \left(2\int_0^{f_h} S_\phi^{\otimes}(f)\frac{\sin^4(\pi f \tau)}{(\pi \nu_0 \tau)^2} df \right)^{1/2}$$



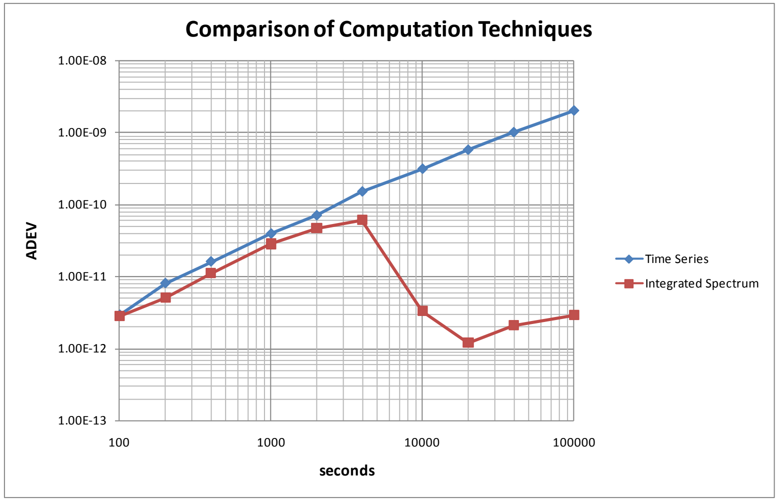
AVAR must be Calculated from the Time Series for τ >> 1

- The Allan deviation may be calculated from the cross spectral density of phase

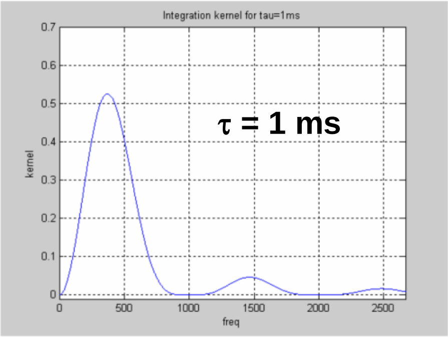
$$\sigma_y(\tau) = \left(2\int_0^{f_h} S_\phi^{\otimes}(f)\frac{\sin^4(\pi f \tau)}{(\pi \nu_0 \tau)^2} df \right)^{1/2}$$

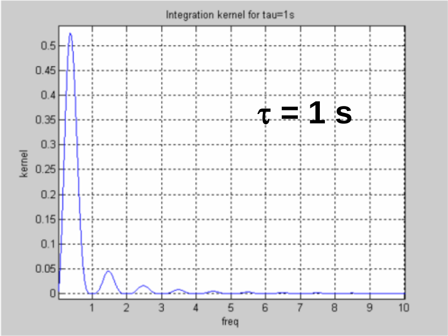
- But for τ large it depends too highly on the first few bins and the integration technique

Behavior of ADEV for computed from the cross spectrum



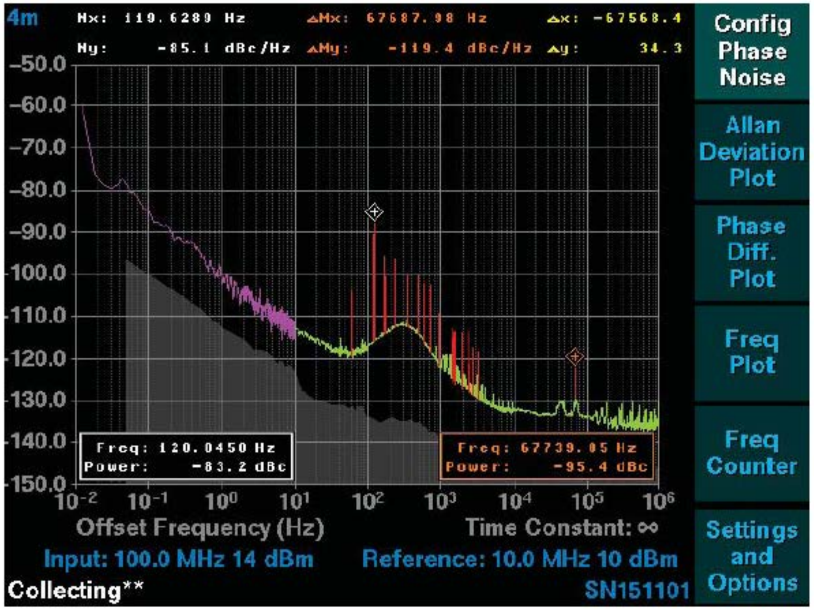
The highly oscillatory behavior and low frequency peaking of the integration kernel where the spectrum estimate is poor makes it impossible to calculate ADEV from the spectrum for long τ





Spurs

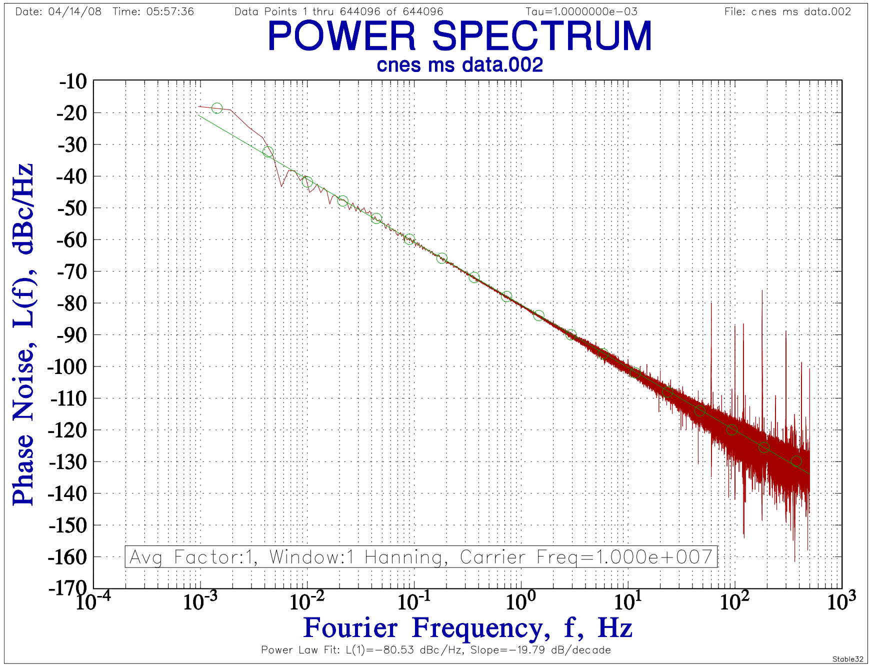
- Real spurs – those that originate in the input and reference sources – must be represented accurately without aliasing
- Internally generated spurs must be identified and removed. These spurs originate from two sources
  - Mixing of input frequencies and sampling frequency
  - Digital synthesis

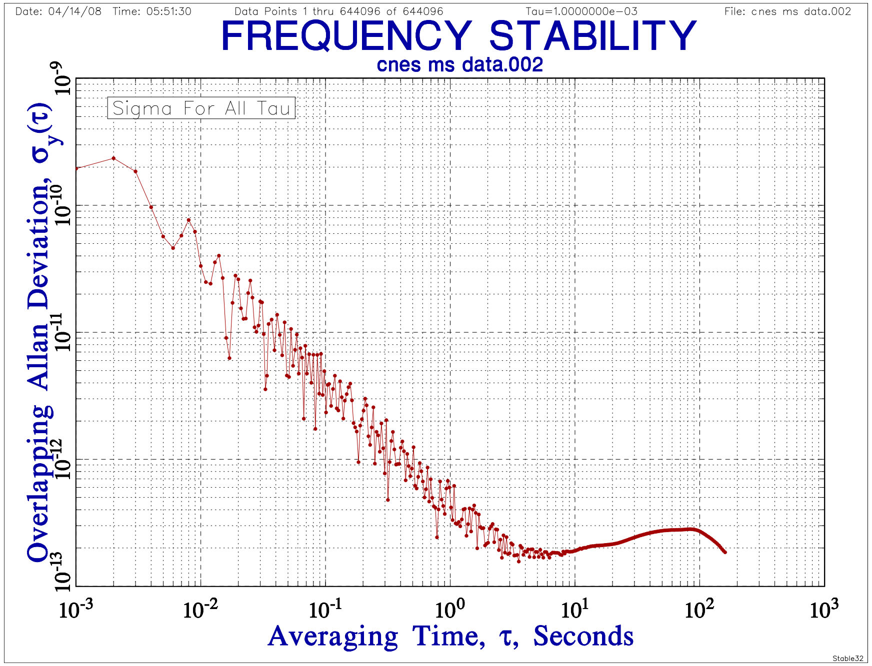


Internal Spurs

- Spurious signals have very high signal-to-noise ratio
- Therefore external spurs have large real part of the cross Spectrum and small imaginary part
  - Their phase angle is close to the real axis
- Internally generated spurs have been found to occur with approximately uniformly distributed phase angle
- Spurs are detected by their signature – main lobe and side lobes with the shape of the window function
- All spurs that lie between 2° and 358° are deleted
  - Each spur deleted removes 0.2% of the bins in a decade span that must be interpolated
  - Approximately 1% of internal spurs cannot be identified and rejected
- Real spurs are not deleted, 50 and 60 Hz spurs are never deleted

Power Line Spurs

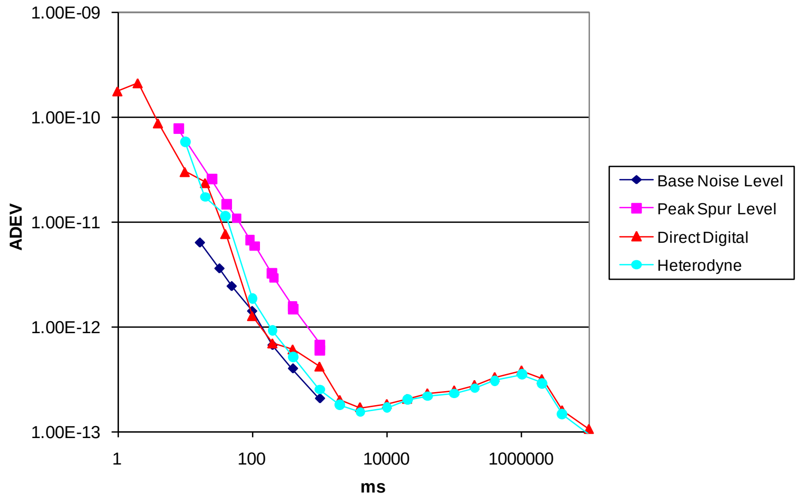




- Peaks occur at half the period of the line frequency and all its harmonics
- Minima occur at multiples of the period of the line frequency and all its harmonics
- It is important that this structure show up at the right τ values

Measured with SP00042 (CNES)

Power Line Spurs may be Incorrectly Analyzed by Heterodyne Test Sets



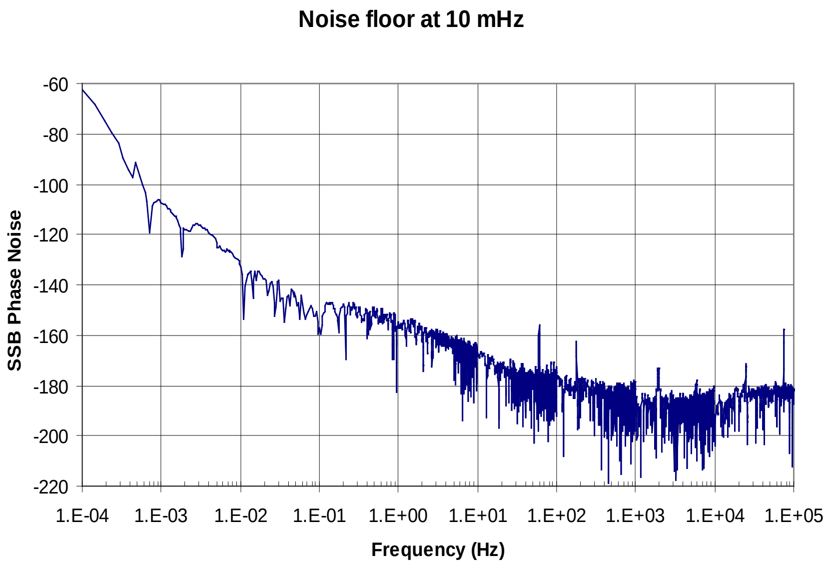
The direct digital test set accurately portrays the line frequency but the heterodyne test set aliases the line frequency and its harmonics to the 0-50 Hz range

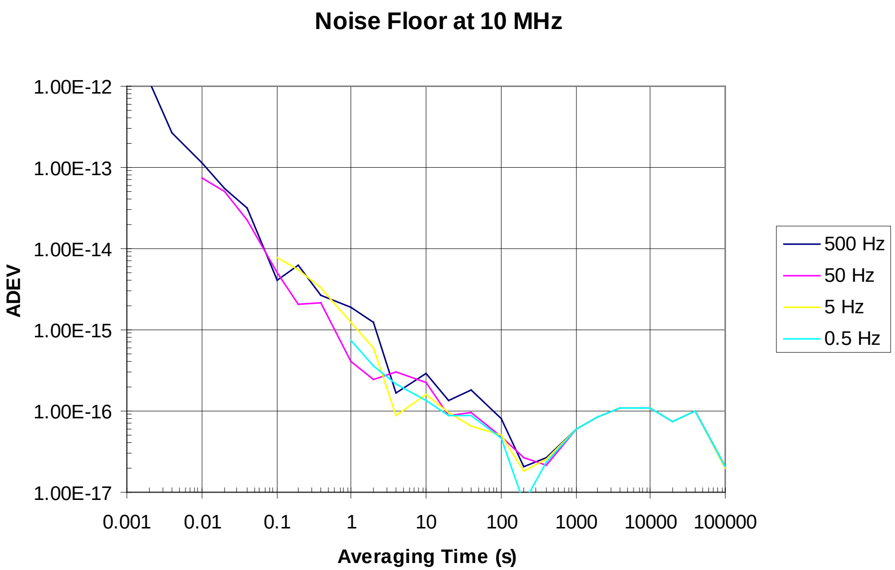
Sparsely sampled ADEV values don't show the underlying structure

Harmonic	US/5120A	5110A Aliased
1	60	40
2	120	20
3	180	20
4	240	40
5	300	0
6	360	40

Harmonic	EU/5120A	5110A Aliased
1	50	50 folded
2	100	0
3	150	50 folded
4	200	0
5	250	50 folded
6	300	0

Direct Digital Noise Floor at 10 MHz

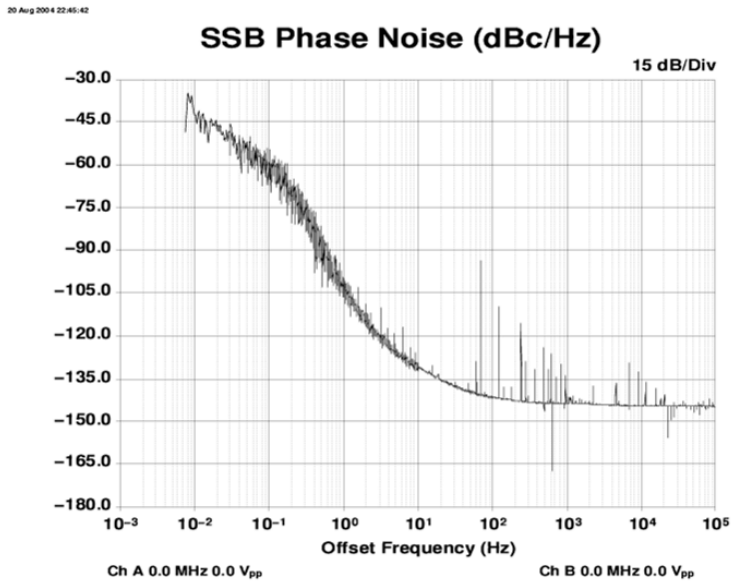




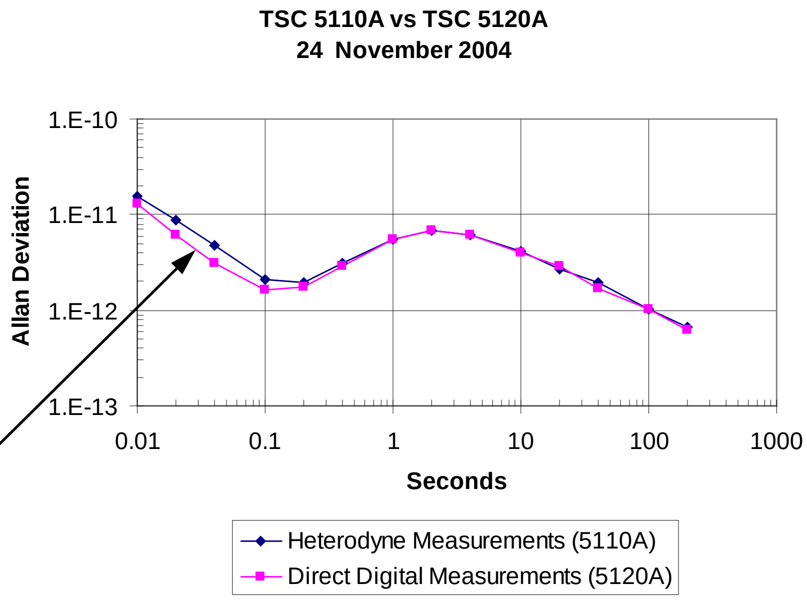
Microsemi

Practical Digital Measurement Examples

Cs vs. Cs
SSB Phase Noise



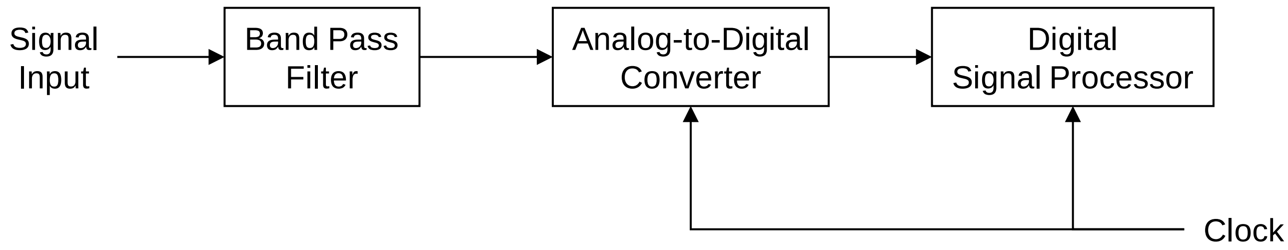
Cs vs Cs
Direct Digital vs Heterodyne



Discrepancy due to coherent
signals at n×60 Hz

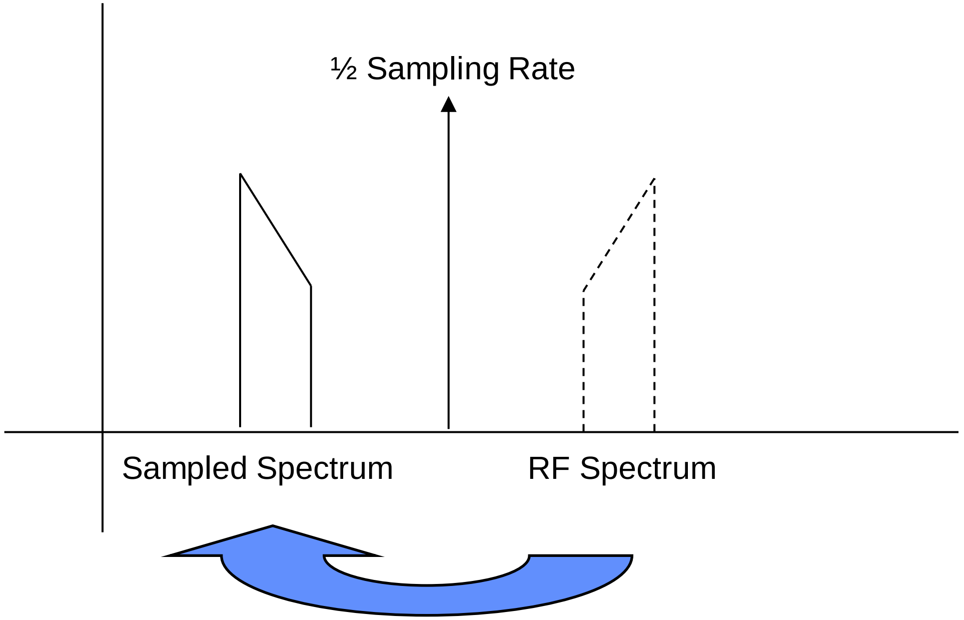
Sub-Sampling Frequency Converter

- Typical high-speed ADC ICs have an RF bandwidth much larger than half the sample clock
- Aliasing can be used to perform frequency conversion without the need for mixers and local oscillators
- Band pass filters are required to ensure that only signals in the desired Nyquist band are converted to base band

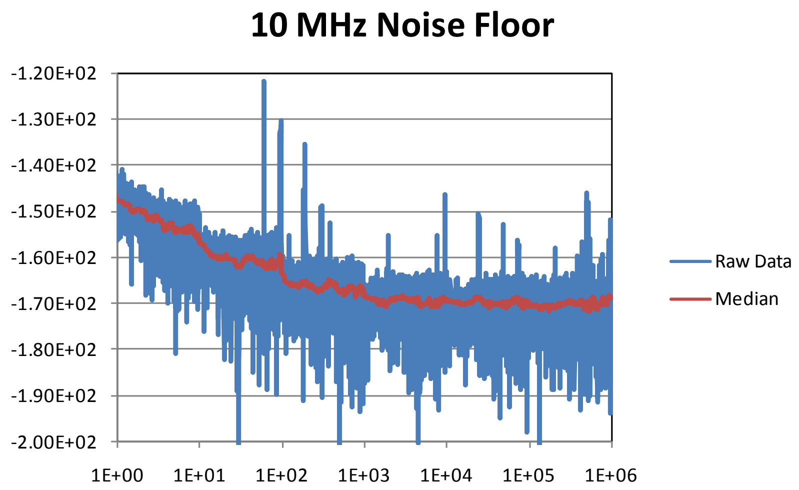


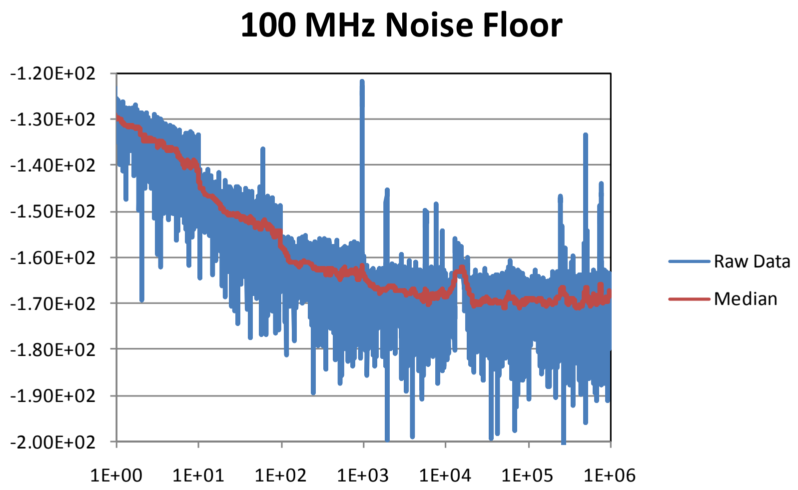
Frequency Conversion by Sub-Sampling

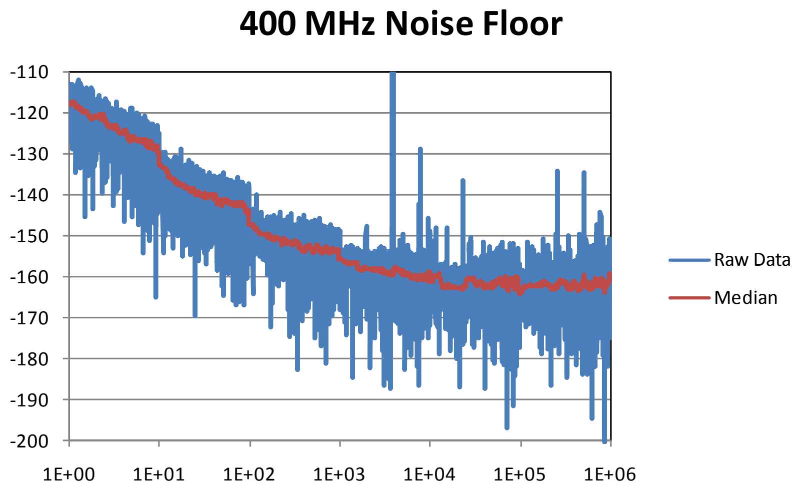
Frequency Range	Spectral Distortion
$N\nu_{\text{sample}}$ to $\left(N+\frac{1}{2}\right)\nu_{\text{sample}}$	None
$\left(N+\frac{1}{2}\right)\nu_{\text{sample}}$ to $(N+1)\nu_{\text{sample}}$	Inverted



Noise Floor at 10, 100, and 400 MHz

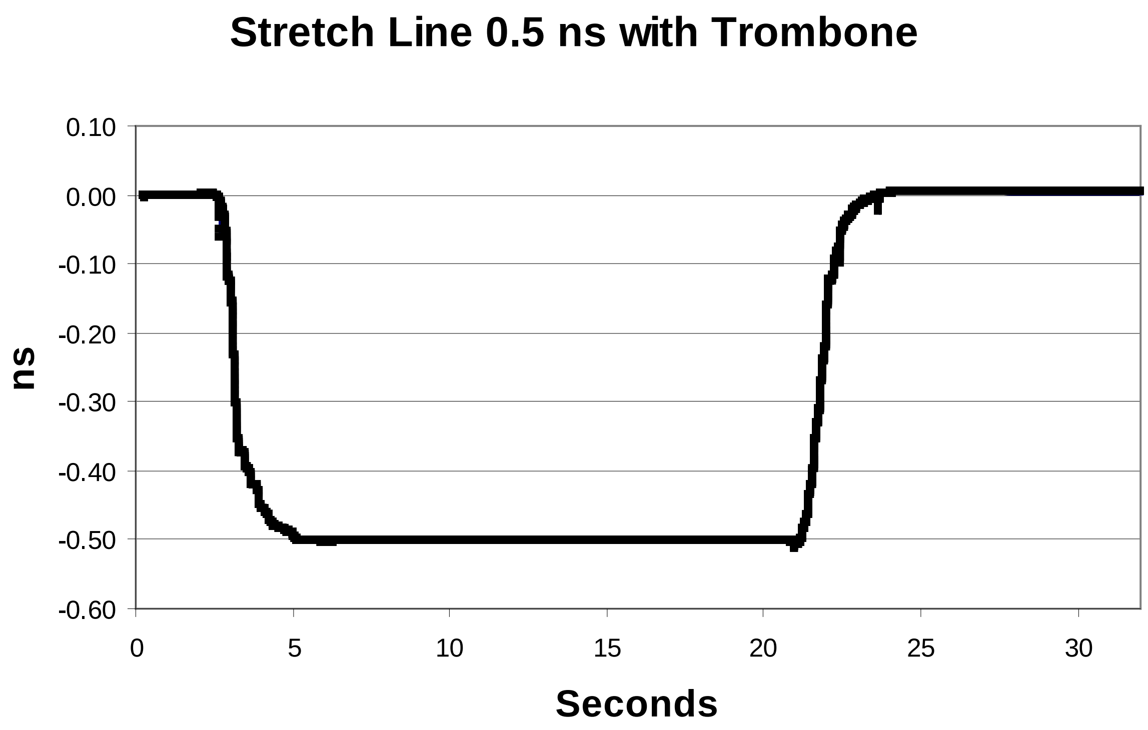






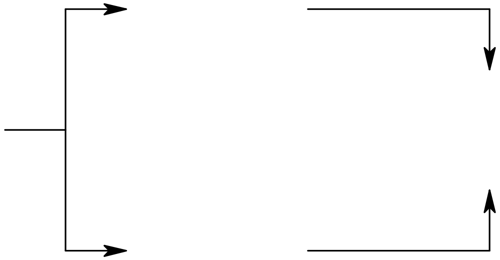
Verification

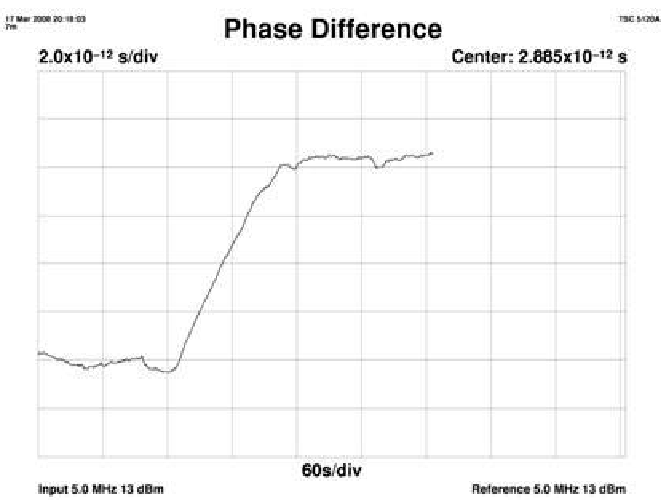
Phase Verification with a Line Stretcher

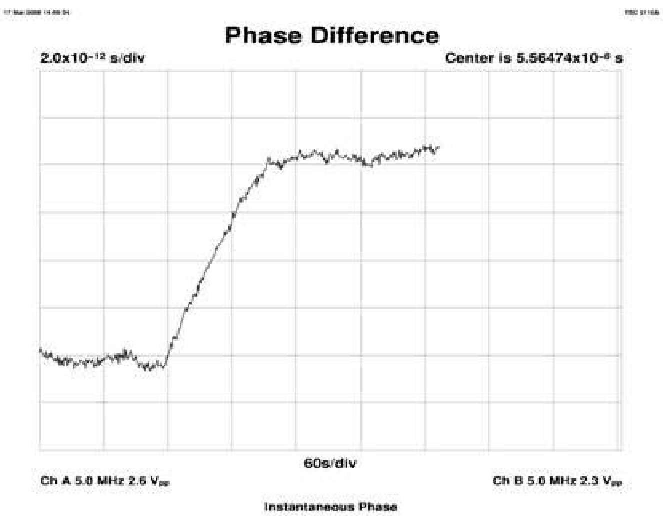


Phase Verification with a Synthesizer

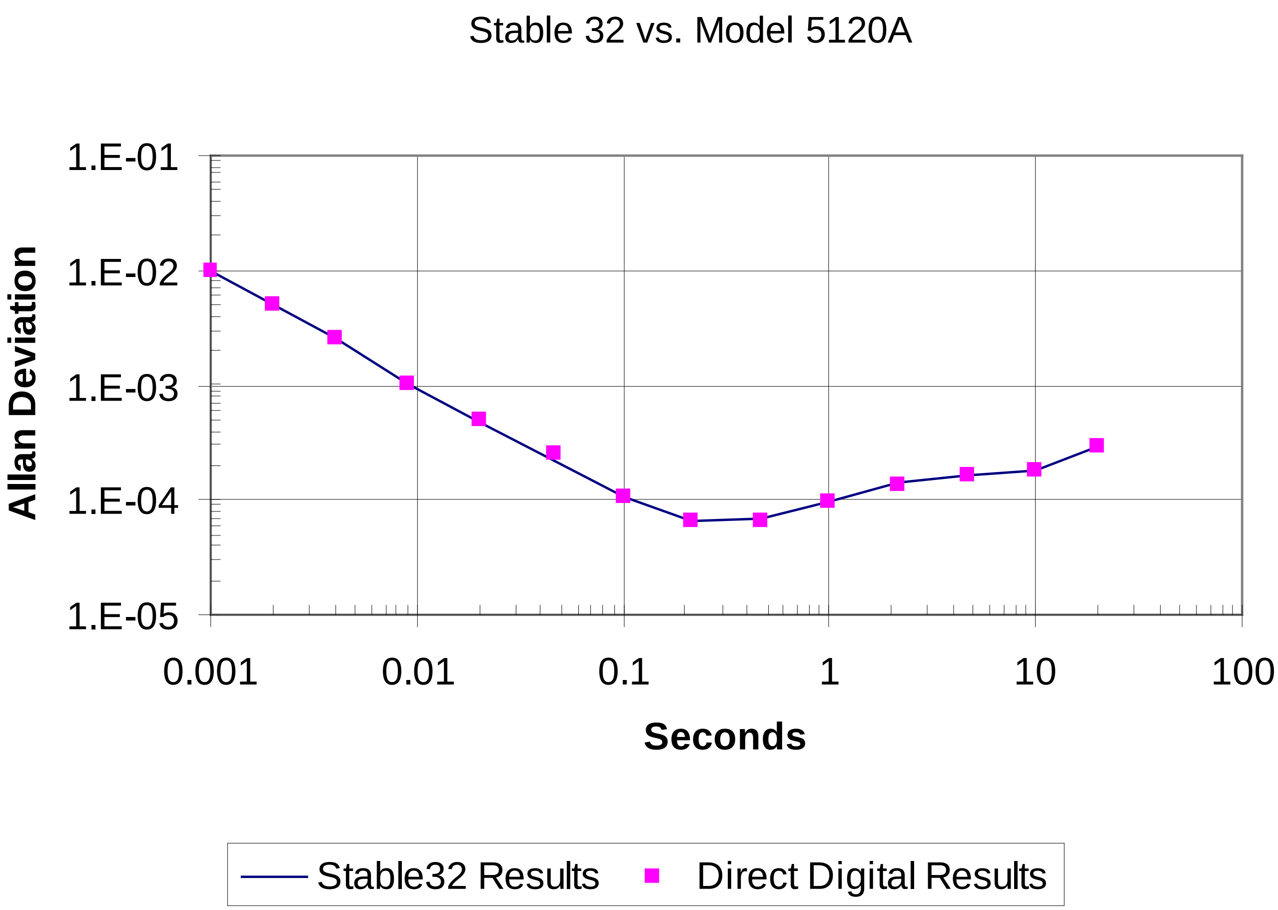
A 9.1 ps phase step was applied with a pair of numerically controlled oscillators (NCO's) by applying a small frequency step with 100 s duration. The measured phase change is about 8.3 ps. The 0.8 ps error is primarily due to noise of the NCO's.



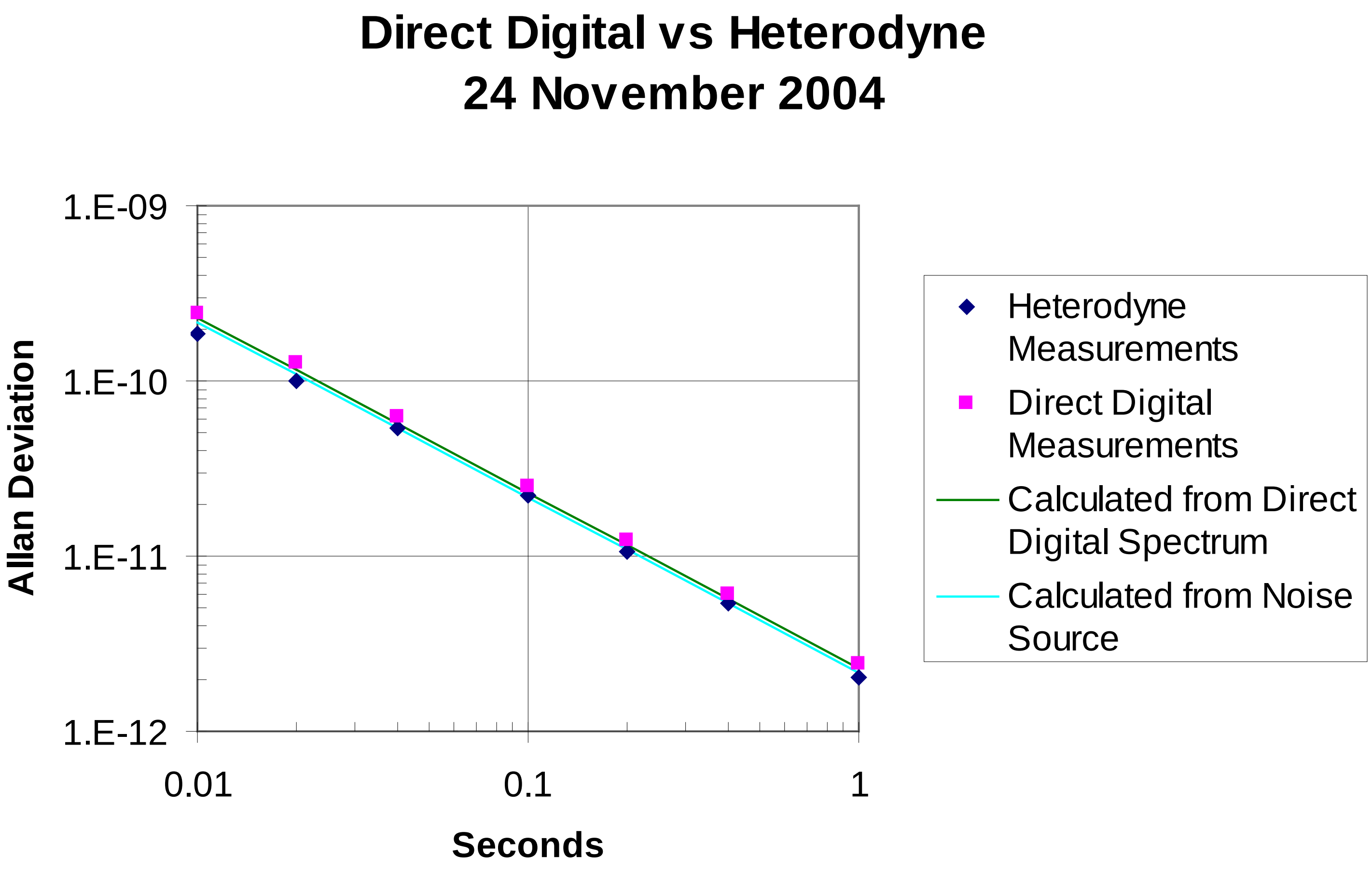




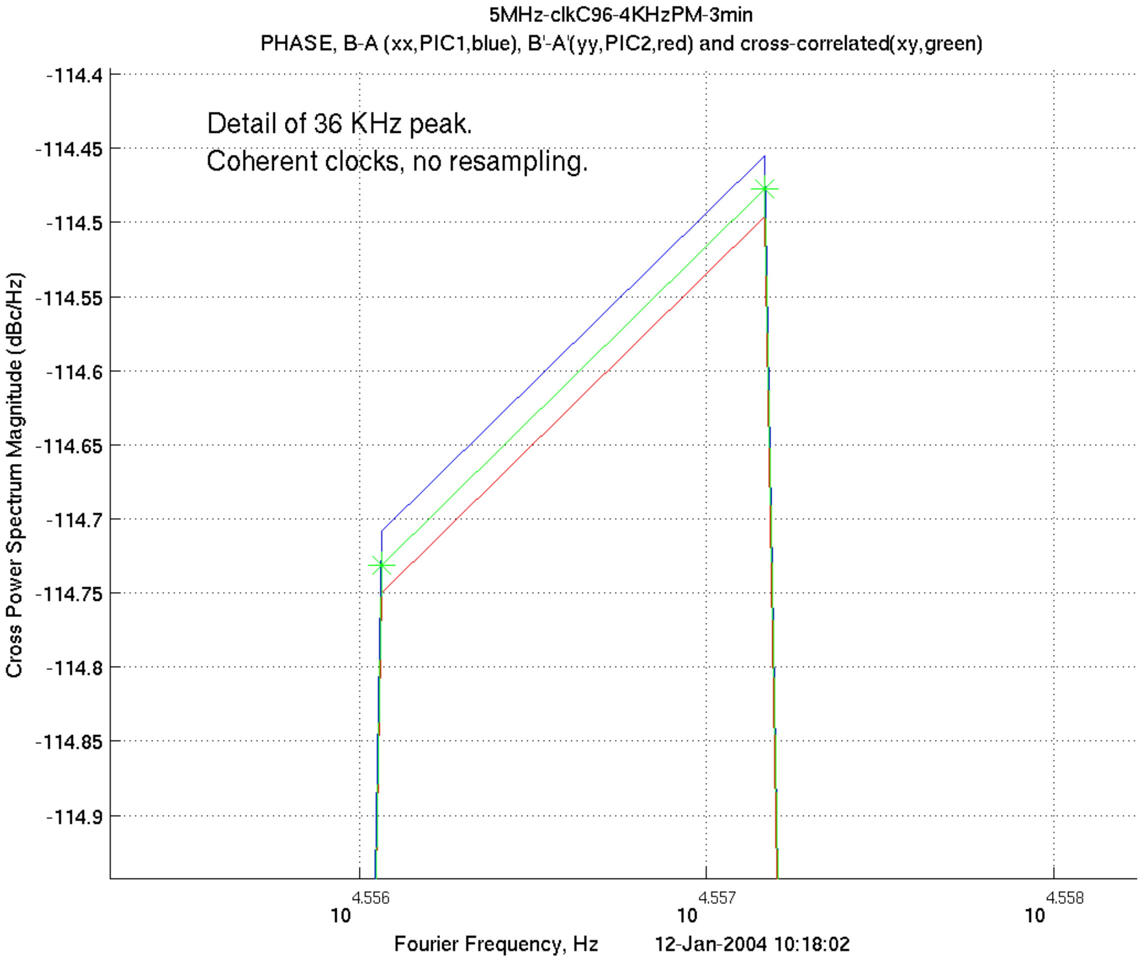
Verification of the ADEV Calculations



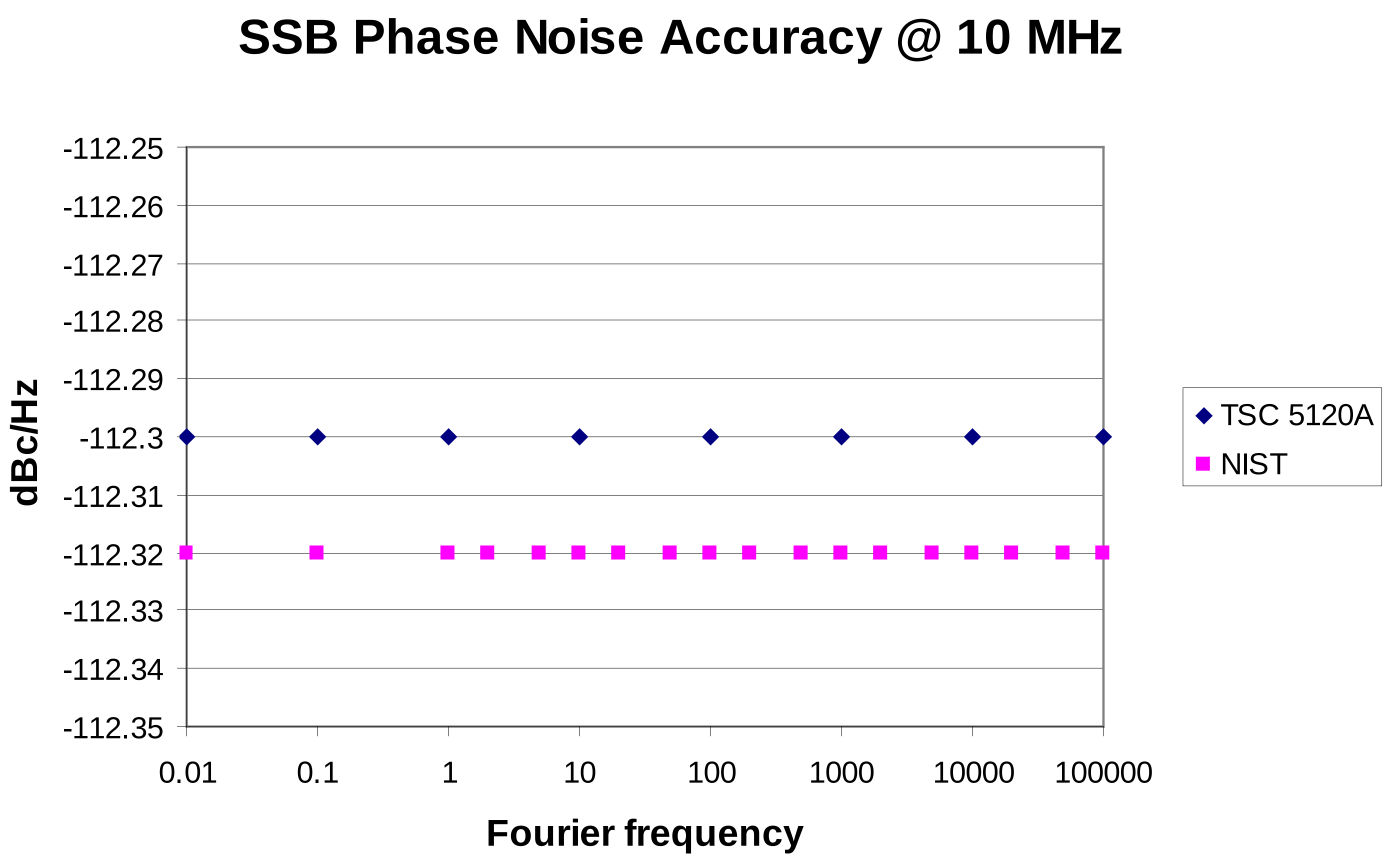
Allan Deviation Verification with White Phase Noise



Processing Verification with a Coherent Signal



Verification with a Calibrated Noise Source



Time and Frequency Domains
Situation Today

Measurements

- Direct digital
  - Maintains epoch
  - 175 dB dynamic range
  - No aliasing

Preserves Carrier

Low noise

Analysis

- Time domain
  - Representation of a signal as a time series
  - Statistics that describe the behavior of a signal over time
  - Very long term
- Frequency domain
  - Representation of a signal as a set of spectral components
  - Very low noise
  - Statistics that describe the spectral content of signals

Microsemi

Concluding Comments

- Digital technology has now improved sufficiently to support converged time and frequency domain measurements to 400 MHz
- Ease of use
- Input and reference can have different frequencies
- Superior accuracy with no user calibration for the setup due to trigonometric phase detector

$$(\omega_L - \omega_C)t - \delta(t) = \arctan\left(\frac{\mathrm{Im}\left[V(t)\right]}{\mathrm{Re}\left[V(t)\right]}\right)$$

- Superior amplitude noise rejection due to ratiometric phase detector
- Simultaneous estimation of phase noise and ADEV
- Measurements below the noise of the internal references
  — Using cross statistics
- Optimized measurement bandwidth