

# Principles of Operation of the Rubidium Vapor Magnetometer

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This paper discusses some of the basic problems involved in designing and using a magnetometer employing optical pumping. Particular attention is given to magnetometers of the self-oscillating type; i.e., those that are analogous to masers in that the resonant properties of the spin system itself are used to sustain continuous oscillation at the resonant frequency. Among the topics treated are amplitude variations with orientation, sensitivity, behavior in extremely weak magnetic fields, and response to rapid field changes.

One of the most important of the practical applications of optical pumping has been to the precise measurement of weak magnetic fields. To this end, several magnetometer systems have been developed using either alkali metal vapor or triplet metastable helium as the rf resonant element. A number of these devices are currently in use for measurement of the geomagnetic field at or near the surface of the earth, and several rubidium vapor magnetometers have been flown in space vehicles. One of these latter instruments successfully measured the interplanetary field out to a distance of 10 earth's radii. This paper discusses the theoretical basis of these Rb space magnetometers, although emphasis is also placed on those problems that arise in operation at the surface magnetic field of about 0.5 gauss. Much of the theory developed here applies to metastable helium magnetometers as well.\*

## I. Fundamentals

The basic theory around which all these instruments are designed was presented in earlier papers by Dehmelt,<sup>1</sup> and by the author and W. E. Bell.<sup>2</sup> All of these systems share the following apparatus in common: A light source producing an intense collimated beam of resonance radiation,<sup>3</sup> a circular polarizer, an absorption

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\* For a more detailed comparison of these two types, see P. L. Bender, *Compt.-rend de 9<sup>e</sup> Colloq. Ampère*, Librairie Payot, Geneva, 1960.

cell containing the vapor to be optically pumped, a radio-frequency coil to produce resonance in the pumped vapor, and a photocell to monitor the transmission of the light. Also under certain circumstances it may be necessary to use a filter capable of eliminating some of the fine structure components of the resonance radiation prior to its entering the absorption cell. A detailed discussion of the conditions under which such a filter must be used will not be undertaken here, except to point out that it depends on the pressure of the buffer gas, if any, that is used to reduce the rate of collisions of the atomic vapor with the walls. The rubidium magnetometers described here normally employ a fairly high pressure buffer gas (3 cm Hg or higher) of neon and a filter is needed to pass the resonance radiation at  $\lambda$  7948 and reject  $\lambda$  7800. An interference filter has been developed for this purpose\* which transmits approximately 80% of the  $\lambda$  7948 radiation for rays whose deviation from the perpendicular is as great as  $15^\circ$ .

The change in optical transmission of the spin system upon application of a resonance rf field has been discussed earlier<sup>1,2</sup> and depends in detail on the direction of the incident light beam relative to the constant magnetic field. The behavior of the spin system can be described by the three variables  $M_x$ ,  $M_y$ , and  $M_z$  of Bloch's equations for a spin system,<sup>4</sup> and the signal present at the photocell is proportional to  $M_n$ , where  $n$  is the direction of the light beam relative to the magnetic field. More specifically, the types of signals that may be observed may correspond either to  $M_z$ , giving rise to a secular change in light intensity, or to  $M_x$  or  $M_y$ , which manifests itself as a high frequency light modula-

\* Manufactured by Spectrolab, Inc., Los Angeles, California.

tion synchronized with the driving rf field. These two types of signals can be readily separated in the electronics following the photocell by virtue of their differences in frequency, and make possible a variety of systems in which the photocell signals can be employed to give information as to the value of the resonant frequency and hence the magnetic field.

It is well known that all magnetometers employing a Larmor resonance have a resonant frequency directly proportional to the applied magnetic field, independent of the angle made by the apparatus relative to the direction of the field. However, the signal amplitude may be a function of this relative orientation. In all of the presently developed optically pumped magnetometers, circularly polarized radiation is employed both as the pumping agent and as the detecting agent. From the symmetry properties under rotation inherent in circular polarization the following angular dependence applies: (1) The secular pumping process varies as  $\cos\theta$ ; (2) detection of the secular light modulation  $M_z$  varies as  $\cos\theta$ ; (3) generation of free spin precession varies as  $\sin\theta$ ; and (4) detection of the Larmor frequency modulation  $M_x$  or  $M_y$  varies as  $\sin\theta$ , where  $\theta$  is the angle between the polar vector defined by the polarized light and the magnetic field  $H_0$ . The types of angular dependence that may be observed in the magnetometer, as a system, may therefore depend on  $\sin^2\theta$ ,  $\cos^2\theta$ , or  $\sin\theta$ ,  $\cos\theta$ , depending on what combinations of pumping and detecting mechanisms are used. Now, for space exploration purposes, one has to assume that the light beam vector may have an equal probability of being in any position relative to the magnetic field. If the signal  $S$  is defined as  $S = 1$  when the instrument is in its most favorable orientation, then we can calculate the probability  $(dP/dS)\Delta S$  that the signal will have an amplitude between  $S$  and  $S + \Delta S$ . It is calculated from the following:

$$dP(\theta)/d\theta = \sin\theta,$$

then

$$\frac{dP}{dS} = \frac{dP(\theta)/d\theta}{dS(\theta)/d\theta}$$

and the right-hand side is solved in terms of  $S$ . The result is

$$\frac{dP}{dS} = \frac{1}{2\sqrt{S}}$$

for  $S = \cos^2\theta$  or  $S = \sin\theta \cos\theta$ ;

$$\frac{dP}{dS} = \frac{1}{2\sqrt{(1-S)}}$$

for  $S = \sin^2\theta$ .

These results are plotted in Fig. 1. All of the devices discussed here follow curve A and due allowance must therefore be made for the high probability of

the instrument having a relatively unfavorable orientation and weak signal. Until recently, no practical

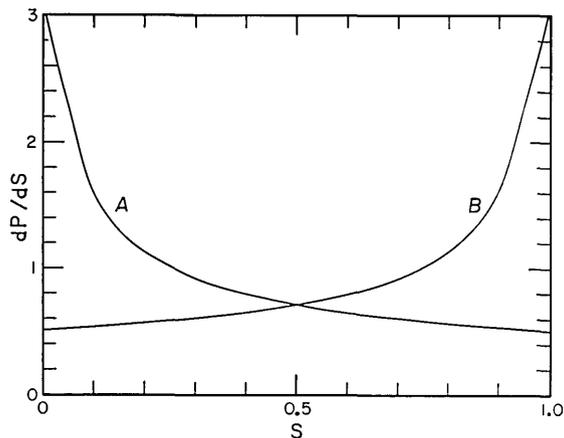


Fig. 1. Differential probability of obtaining a signal of amplitude  $S$ . Curve A:  $S = \cos^2\theta$  and  $S = \sin\theta \cos\theta$ . Curve B:  $S = \sin^2\theta$ .

instruments had been suggested that might have a response given by curve B. However, the recent observation of optically driven spin precession,<sup>5</sup> which has a  $\sin^2\theta$  dependence, makes such a magnetometer feasible<sup>6</sup> and suggests a close examination of the behavior and requirements of such magnetometers. This is not attempted in this paper.

## II. Possible Systems

Three types of systems have been developed into practical magnetometers.

The first of these<sup>7</sup> is shown in Fig. 2, and employs the

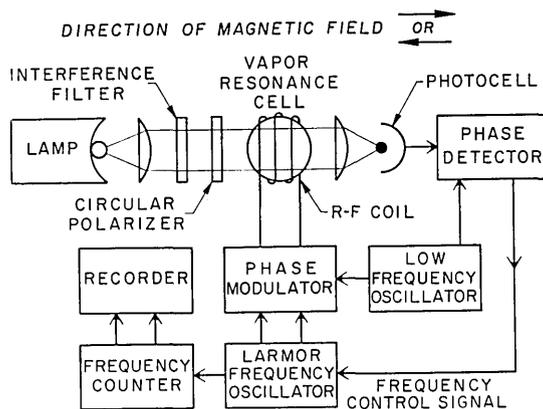


Fig. 2. Block diagram of magnetometer employing  $M_z$  signal and phase detector to stabilize a local oscillator at the Larmor frequency. Minor variations of this scheme may employ magnetic field sweep, low-frequency offset oscillators, or servo loops responsive only within a certain range of frequencies.

$M_z$  signal. A small frequency modulation is applied to the local rf oscillator to sweep the rf back and forth through resonance. The photocell signal then has strong components at even harmonics of the modulation frequency if the sweep is centered about the resonance. Any deviation gives rise to a fundamental component which is phase detected and used to servo the tracking rf oscillator. The angular dependence of the signal varies as  $\cos^2\theta$ . This type of system does not place severe restrictions on the performance of the photocell or its amplifier, and it employs a type of electronic detection system that is well known in the art of magnetic resonance. It has been widely employed to demonstrate the operation, in principle, of optically pumped magnetometers. However, for airborne and space applications, where power and weight requirements are at a premium, this type of system does not possess the advantages of extreme simplicity and wide dynamic range that are characterized by the next two systems to be described. Accordingly all of our present work in Rb vapor magnetometers is concentrated on these two latter types of systems. The remainder of this paper will be concerned primarily with them.

The next system to be described will be designated as the "single cell self-oscillator" and is shown in Fig. 3.

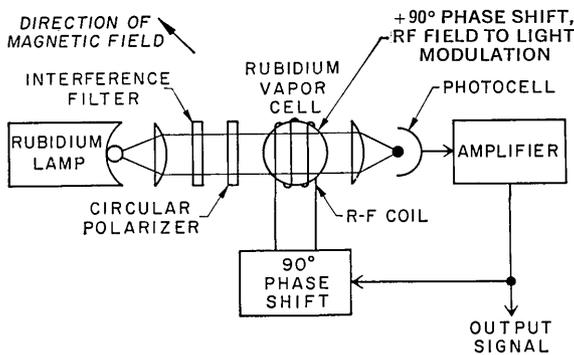


Fig. 3. Self-oscillator type of magnetometer employing a single absorption cell.

Here the photocell observes the signal corresponding to  $M_x$ , which is at the driving radio frequency, and whose phase relative to the driving rf field varies near resonance in much the same way as the phase of the circulating current in a tuned resonant circuit. The photocell signal is fed back through suitable amplifiers and phase shifters to the rf coil and maintains a closed loop through which information can be transmitted only within the resonance line width. This system will oscillate spontaneously at the resonant frequency if the

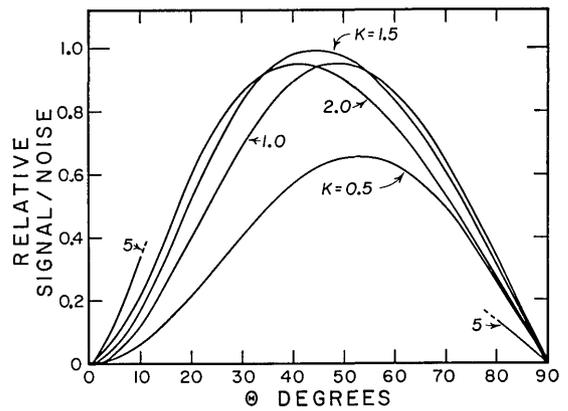


Fig. 4. Signal amplitude or signal-to-noise ratio of the self-oscillator as a function of  $\theta$ . These curves differ from the simple  $\sin\theta \cos\theta$  dependence in that they include the effect of variation of the perpendicular component of the rf field.  $K = \gamma H_a (T_1 T_2)^{1/2}$  where  $2H_a$  is the peak value of the alternating rf field along the coil axis.

gain of the loop is unity and the sum of all phase shifts in the loop is zero. In order to reduce the number of axes of symmetry to a minimum, the rf coil is wound coaxial with the light beam so that only the one angle  $\theta$  suffices to describe all variations in performance such as are shown in Fig. 4. (If the rf coil axis were perpendicular to the light beam, then the phase shift would vary with rotation of the instrument about the light beam axis.) The component of rf field  $H_1$  perpendicular to  $H_0$  defines the  $x$ -direction, and correspondingly the alternating signal observed by the photocell always corresponds to  $M_x$ , rather than to  $M_y$  or some linear combination. Bloch's equations predict a  $90^\circ$  phase shift between  $H_1$  and  $M_x$  at exact resonance, and this is the reason for the  $90^\circ$  phase shifter.

The system just described has the following disadvantage: In the angular dependence of the signal as given by  $\sin\theta \cos\theta$  or in Fig. 4, the change in sign which occurs at  $\theta = 90^\circ$  must be taken literally. It corresponds to the fact that if the instrument is properly phased for  $0 < \theta < 90^\circ$ , then it is wrongly phased for  $90^\circ < \theta < 180^\circ$ . An engineering solution to this problem has been obtained by including, in the feedback loop, a "signal searcher" which periodically interchanges the connections of the rf coil until oscillation is established. However, a much more elegant solution is that of the third system to be described, shown in Fig. 5, which will be designated as the "two-cell self-oscillator system." This system consists of two identical optical systems in which one photocell output is used to drive the rf coil of the complementary absorption cell. It may be described most simply by saying that each optical system acts as a  $90^\circ$  phase shifter for the other. Here the sum of the phase shifts of the entire feedback loop can be made zero without

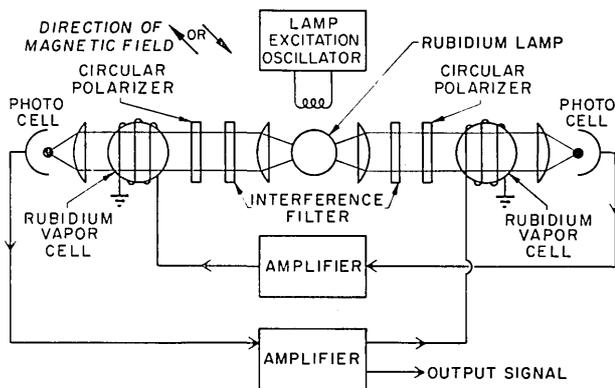


Fig. 5. Self-oscillator employing two absorption cells. If the two optical systems are placed back-to-back, as shown, then the two circular polarizers should be of the same sense (e.g., right-handed) to preserve symmetry upon 180° rotation.

insertion of electronic phase shifters and the device has complete symmetry for 180° rotations, so that it will oscillate spontaneously in all directions except  $\theta = 0$  or 90°.

Both of the above types of instruments have been flown in space vehicles.

### III. Theoretical Sensitivity

For our purposes we are concerned with two aspects of sensitivity: (a) the smallest absolute value of the field that can be measured by the apparatus (important for interplanetary field measurements) and (b) the smallest increment in magnetic field, at some reasonable ambient level, which can be measured.

The smallest absolute value of field that can be measured by any Larmor resonance apparatus is roughly equal to the rf resonance line width expressed in magnetic field units, since at smaller values the response is no longer that of a resonant system. This point is elaborated on in Section IV, where the problems involved in reaching the limit are defined more precisely. The total line width  $\Gamma$  of the resonance is given in general by

$$\Gamma = \Gamma_r + \Gamma_t$$

where  $\Gamma_r$  represents the effect of absorption of resonance radiation and  $\Gamma_t$ , due to thermal relaxation, is the minimum line width that is obtainable in the limit of zero light intensity (and therefore in the limit of zero signal-to-noise ratio!). For the alkali vapors,  $\Gamma_t$  has been found<sup>1</sup> to be of the order of 0.03 cps. For Rb<sup>87</sup>, which has as high a  $g$ -factor as can be found in any of the alkali isotopes, the gyromagnetic ratio is 700 kc/s per gauss so that the above number corresponds to a minimum magnetic field of less than  $10^{-7}$  gauss. As a practical matter, however, including line broadening from the light, the minimum measurable field appears

to be about  $2.0 \times 10^{-5}$  gauss. Metastable helium, whose gyromagnetic ratio is 2.8 Mc/s per gauss, appears to have a minimum line width of about  $10^3$  cps corresponding to  $40 \times 10^{-5}$  gauss.

The sensitivity of the instrument for small increments of field depends in part upon line width, and in part upon actual signal-to-noise ratio for the resonance signal itself. Because of the success of theoretical signal-to-noise calculations in nuclear and electron spin resonance, it is attractive to attempt the same sort of calculation for an optically pumped system. Unfortunately, the problem in this case is made difficult by the large number of variables whose effect on performance is difficult to assess experimentally, much less theoretically, and because comparison with a standard is not available. For a given configuration we might proceed as follows. If we start with the assumption that the light source is perfectly incoherent, and that the photocell has unity quantum efficiency and zero dark current, then the noise current  $N$  is given by

$$N = (2e^2 n B)^{1/2}$$

where  $e$  is the electronic charge,  $n$  is the number of photons per second, and  $B$  is the effective bandwidth; in this case  $B = \Gamma$ . Since  $\Gamma_r$  is proportional to  $n$ , we have

$$N \propto (\Gamma \Gamma_r)^{1/2}.$$

The relative signal current,  $S$ , is calculated in formula (10a) of I and can be written as

$$S \propto \Gamma_r^2 / \Gamma.$$

The sensitivity of the instrument for small changes in field in its optimum orientation is then given by

$$\frac{S}{N\Gamma} = \frac{\Gamma_r^{3/2}}{(\Gamma_r + \Gamma_t)^{5/2}}.$$

This has a maximum at  $\Gamma_r = \frac{3}{2}\Gamma_t$ , suggesting operation with narrow lines and weak light. In practice, however, other sources of noise are also present and it is usually found desirable to operate with the highest possible light intensity consistent with other requirements. Typical operating conditions, for moderately strong magnetic fields and illumination, are approximately as follows: Average photocurrent, about  $1.0 \times 10^{-4}$  amp; noise,  $N = 1.65 \times 10^{-9}$  amp; signal, about 1% of average photocurrent or about  $1.0 \times 10^{-6}$  amp; line width at this light intensity, about 100 cps. The minimum detectable change in frequency,  $N\Gamma/S$ , is then about 0.025 cps corresponding to a field change of  $3 \times 10^{-8}$  gauss for Rb<sup>87</sup>. This is comparable to experimentally determined results and suggests that photoelectron shot noise is indeed the limiting factor for this particular configuration. At lower light intensities, dark-current or amplifier noise seems to predominate. This might be avoided by increasing the

diameter of the optical system from the present 5 cm, while maintaining good optical geometry so that the size and brightness of the image on the photocell is comparable to that now available. An increase in diameter, however, is incompatible with size and weight limitations for space applications.

#### IV. Perturbations from the Larmor Resonance Condition

Up to now, we have specified that the oscillation frequency of either of the two self-oscillating magnetometers is actually the Larmor (angular) frequency  $\omega_0 = \gamma H_0$ . However, we must be concerned about the possibility of deviations from this condition, for two reasons. First, we demand that the magnetometer measure field increments much smaller than the resonance line width. Since the usual first-order perturbation theory leading to a description of resonance effects is good only to within orders of magnitude of the line width, it is therefore apparent that we must look for higher order effects. Second, operation of the magnetometer in extremely weak fields may bring us close to the condition where the spin system is no longer recognizable as a resonance system, and in that case we must be concerned as to exactly how far we can push the operation of the system toward zero magnetic field. We shall consider the effect of variations of phase in the feedback loop, and then go on to effects intrinsic to the spin system that have not been discussed so far.

#### Phase Relationships

In the vicinity of resonance, the amplitude of the received signal  $M_x$  is approximately constant (it is maximum by definition), but the phase varies as  $\text{ctn}^{-1}(T_2\Delta\omega)$ , where  $1/\pi T_2$  is the line width (in ordinary frequency units) and  $\Delta\omega$  is the deviation from resonance.\* It is clear from this that even with the narrowest lines that are conveniently available, say 15 to 100 cps, the phase of the feedback loop and of the received signal  $M_x$  must be held constant within  $0.1^\circ$  or better if the magnetometer is to retain its potential sensitivity. Most of the problems that are experienced in practice, in maintaining correct phase relationships, are electronic ones that will not be of concern here. We will mention one additional problem—that of effects of misorientation of the mean axis of the rf coil. If the coil axis is not exactly parallel to that of the light beam, then the coaxial symmetry which was discussed earlier is partially destroyed and orientation effects can show up in the form of spurious phase shifts. The maximum possible shifts are shown in Fig. 6. The figure shows that serious effects occur only when  $\theta$  is small; however,

\* From this point on, the mathematical notation is identical to that employed in I except that  $S_1, S_2$  are replaced by the more conventional symbols  $T_1$  and  $T_2$ .  $T_1, T_2$  are relaxation times including lifetime-shortening effects of the light.

under such conditions even a small misorientation angle can give rise to a large error. Special care is therefore required in winding the rf coil about the absorption cell and maintaining it rigid with respect to the external supports, also in maintaining an optical system with well-defined axis of symmetry despite the broadness of the source.

#### Higher Order Terms in Bloch's Equations

In I there was developed a solution to the equations of motion of a spin- $1/2$  optically pumped system based on Bloch's equations. It was pointed out there that, although the equations held strictly only for spin- $1/2$  particles, it was hoped that they might find more general application for the more complicated spin system which actually represents a Rb atom. Experience under a wide variety of conditions has shown that these equations do indeed give a quite accurate representation of the behavior of the signals, except for some specific instances where the hyperfine coupling results in a multiplet structure. These situations are dealt with later. We are concerned here with those deviations from the normal resonance conditions that can be predicted within the scope of Bloch's equations. These effects are particularly important in very weak fields.

In I we developed a Fourier expansion of Bloch's equations in terms of components at the driving frequency and its harmonics. The expansion is as given overleaf

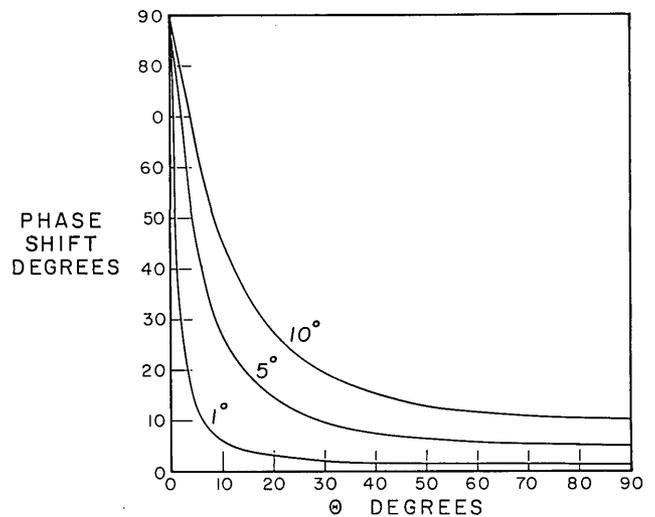


Fig. 6. Phase shift in the photocell signal as a function of orientation angle,  $\theta$ , shown for three misorientation angles of the rf coil axis relative to the optical axis.

$$M_x + iM_y = \sum_{n=-\infty}^{\infty} f_n e^{in\omega t},$$

$$M_z = \sum_{n=-\infty}^{\infty} m_n e^{in\omega t}.$$

Of the various coefficients,  $f_1$ ,  $f_{-1}$  and  $m_1 (= m_{-1}^*)$  are at the driving frequency itself and can therefore all take part in the signal contained in the feedback loop. The other terms can be considered as perturbations contributing to the final solution, and of these only the terms with  $|n| \leq 2$  will be different from zero except under the most extreme conditions. Solution of Eqs. (14a) and (14b) of I for a weak  $H_1$  field ( $m_0 \approx M_0$ ) gives the following for  $M_{osc}$ , the magnetic moment oscillating at the driving frequency. With the phase chosen by  $H_x = 2H_1 \cos \omega t$  we have:

$$M_{osc} = M_x \sin \theta + (M_z)_1 \cos \theta,$$

$$M_x = \gamma H_1 M_0 \left[ \frac{\omega_0 + \omega}{T_2^{-2} + (\omega_0 + \omega)^2} + \frac{\omega_0 - \omega}{T_2^{-2} + (\omega_0 - \omega)^2} \right] \cos \omega t$$

$$- \frac{\gamma H_1 M_0}{T_2} \left[ \frac{1}{T_2^{-2} + (\omega_0 + \omega)^2} - \frac{1}{T_2^{-2} + (\omega_0 - \omega)^2} \right] \sin \omega t,$$

$$(M_z)_1 = \frac{2\gamma H_1 T_1 T_2 \omega_0 M_0' (\cos \omega t - \omega T_2 \sin \omega t)}{(1 + T_1^2 \omega^2)(1 + T_2^2 \omega^2)}.$$

The  $M_x$  part of the equation consists of the usual resonance expression plus additional terms that take into account the fact that the alternating rf field really contains two frequencies,  $+\omega$  and  $-\omega$ , and each of these must act independently upon the spin system. The extra terms cannot be ignored in weak fields. They give rise to a deviation in the apparent Larmor frequency, in that the frequency for which there is a  $90^\circ$  phase shift is given by

$$\omega = [(1/T_2)^2 + \omega_0^2]^{1/2}.$$

The effect may be of importance even when the field differs from zero by several line widths. It represents, however, a fixed correction that can be included if  $T_2$  is known.

Simultaneous with the shift in phase relationships is a change in signal amplitude. The signal amplitude at the frequency at which there is  $90^\circ$  phase shift is proportional to  $\omega_0 [(1/T_2)^2 + \omega_0^2]^{-1/2}$  if  $H_1$  is held fixed and more nearly proportional to  $\omega_0^2$  if  $H_1 < H_0$ . It is apparent from this that one can get self-oscillation in fields much smaller than a line width if signal-to-noise is available; however, it is also clear that there will not be much sensitivity for variations of the "true" Larmor frequency in this case.

The effect of the  $(M_z)_1$  term is to introduce additional spurious phase and resonance amplitude information, which varies in a complicated way with  $\theta$  and with frequency. No detailed attempt has been made to study the effect of this perturbation analytically, except to note that it is small for magnetic fields greater than about one line width.

The effect of an rf field not exactly perpendicular to  $H_0$  is to introduce, in the above equations, a correction term to  $M_0$  proportional to  $(1 + \omega_0^2 T_2^2)^{-1}$ . However, it does not change the above results appreciably.

The Bloch-Siegert effect<sup>8</sup> is a well-known effect depending upon higher order perturbation terms involving  $H_1$ . It can be obtained in the above formalism by including  $m_2$  and  $m_{-2}$  in the closed solution for  $f_{-1}$  and gives an apparent decrease in resonant frequency given by

$$\delta\omega = \gamma H_1^2 / 4H_0.$$

Its effect can be maintained moderately small, although perhaps not negligible, by always maintaining  $H_1 \ll H_0$ .

To summarize the above results, if the oscillation frequency of the self-oscillator is greater than the line width, then it represents the true Larmor frequency with possible small and known corrections. In a given field (such as 0.5 gauss) these corrections are the smaller the narrower the line width. For fields less than about one line width, however, the oscillation frequency depends in a complicated way on orientation angle and other variables but does not in general become less than the line width. An output frequency close to the line width is therefore suspect unless the exact operating conditions of the instrument are known.

## Effects of Line Structure

The hyperfine splitting in the alkali atoms gives rise to an incipient Back-Goudsmit effect, which is manifested as a splitting of the resonance line into  $2F + 1$  components, the separation between components depending upon the hyperfine constant and being proportional to  $H_0^2$ . It turns out that for virtually all stable alkali isotopes, this splitting is negligible for  $H_0 < 0.1$  gauss, but is relatively large at  $H_0 = 0.5$  gauss. Table I gives the details of this structure for a field of 0.5 gauss.

**Table I. Splitting of the Zeeman Resonance in a Field of 0.5 Gauss<sup>a</sup>**

Isotope	Number of lines $2I + 2$	Separation between adjacent lines		$\gamma_n H_0 / \pi$ for $H = 0.5$ (cps)
		(cps)	( $10^{-5}$ gauss)	
Na <sup>23</sup>	5	138	19.8	1126
K <sup>39</sup>	5	531	76	199
K <sup>41</sup>	5	1005	144	109
Rb <sup>85</sup>	7	36	7.7	411
Rb <sup>87</sup>	5	36	5.2	1393
Cs <sup>133</sup>	9	6.7	1.9	558

<sup>a</sup> Only the hyperfine level  $F = I + J$  is considered here; the  $F = I - J$  level usually gives much weaker signals, for reasons that are not entirely clear. The column  $\gamma_n H_0 / \pi$  gives the separation between the centers of the  $(I + J)$  and  $(I - J)$  patterns.

In the presence of this structure, the designer of the magnetometer can do one of two things. He can either attempt to operate on only one of the individual components, hoping that the presence of the others will not perturb the operation appreciably, or he may attempt to operate under conditions of a relatively broad line in which case the structure is almost completely submerged under the over-all resonance. Magnetometers designed according to the first principle have been made to operate in the laboratory, but operation by the second method is much to be preferred in most cases, since the line width of a hundred cycles or so is compatible with good signal-to-noise ratio.

The line shape obtained by smearing over an unresolved structure is bound to be unsymmetrical in a rather complicated way, depending on the matrix elements for the various individual transitions and other considerations. However, this asymmetry should invert itself when the magnetic field is reversed in direction, since this is equivalent to interchanging the signs of all magnetic quantum numbers. It is apparent that a single-cell self-oscillating magnetometer in  $H_0 = 0.5$  gauss may therefore exhibit a fairly large orientation dependence as  $\theta$  is varied between  $0^\circ$  and  $180^\circ$ . Fortunately the double-cell self-oscillator magnetometer avoids this situation, as it does many other situations of asymmetry, provided that the sense of circular polarization of the light beams is chosen to be opposite as observed by a single observer (Fig. 5). In this way, the two components of the dual oscillator tend to "pull" themselves together to a common median frequency, which is also the center of symmetry of the two line shapes. The requirement of perfect mirror symmetry, however, requires also that the behavior of the system, i.e., gas cell absorption, photocell efficiency, amplifier gain, etc., be identical in both systems.

## V. Response to Rapid Changes in Magnetic Field

One of the interesting characteristics of the self-oscillator magnetometer is its ability to respond to rapid and large excursions in magnetic field value. In general, a freely precessing spin follows faithfully all instantaneous changes in field; questions then arise as to (a) whether these changes can be detected in the output, and (b) whether the effect of the feedback loop has any effect on the instantaneous motion. With regard to the first question, the output will be a faithful representation of the input provided that the amplifiers are of sufficient bandwidth to accommodate all of the various frequency components generated during the frequency excursion. As for the second question, it can again be shown that the feedback loop will remain tightly locked as long as the signals are within the amplifier bandwidths; however, it is not necessary that this be so. As long as the time interval of the ex-

ursion is less than the  $T_2$  of the precessing spin system, then the rf in the feedback effectively has no driving force on the spins during this time interval, and the spins are in free precession as if the rf were not there. (For longer excursion times the bandwidth required, which is inversely proportional to the excursion time, is certainly adequate.)

We will now support the statement that the value of the rf field in the feedback loop is never so great as to affect the spin precession in a time less than  $T_2$ . The time required for the rf field to interact with the spin system is approximately  $(\gamma H_1)^{-1}$ . If one attempts to operate the oscillator with  $\gamma H_1 T_2 > 1$ , one finds that a very undesirable amplitude variation or "squegging" occurs, in which the oscillation alternately builds up and quenches at a rate roughly proportional to  $H_1$ . The effect was first observed experimentally and then it was found, on a theoretical basis, that effects of this sort can be expected in all quantum resonance systems involving feedback, as for example in masers. A detailed treatment of the problem has been given by Singer and Wang.<sup>9</sup> It is sufficient to remark here only that the response of the spin system with regard to the feedback loop is similar to that of a spin system in a maser (in which case the feedback loop consists of the radiation reaction inside the cavity). Since the "squegging" is very undesirable, gain controls must be introduced in the magnetometer amplifiers to keep the rf field down to a safe value, and this also automatically eliminated rapid transient perturbations of the rf on the spin system.

A more detailed study of the rapid response characteristics of the magnetometer was made by simulating both single- and double-cell magnetometers on an analog computer. In this study the time development was started with the spins in equilibrium with the driving rf field corresponding to continuous oscillation. A transient was then introduced equivalent to making  $H_0$  first slightly negative and then highly positive in a short time interval. An example of such a run is shown in Fig. 7. It can be seen there that the quantity given by  $d\varphi/dt$ , where

$$\varphi = \tan^{-1}(M_y/M_x)$$

does indeed represent a faithful description of the instantaneous Larmor frequency. However, if the received information consisted of either  $M_x$  or  $M_y$  alone, then it might be difficult to resynthesize the exact frequency and phase variations, particularly when the field reverses direction. There is therefore a great advantage in two-phase relative to single-phase output information for this type of situation. In this case, the double gas cell magnetometer has a definite advantage over the single cell magnetometer in that two-phase information is actually available from the outputs of the two photocells.

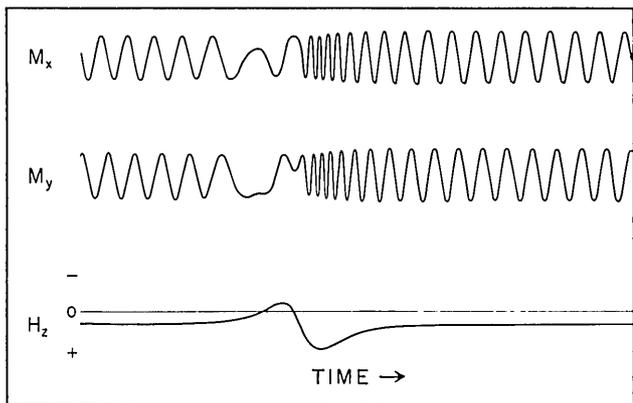


Fig. 7. Analog computer curves showing response of self-oscillator system to relatively large and rapid magnetic field transients.

The above comments pertain to the situation in which the magnetic field transient consists of a vector parallel to the average magnetic field  $H_0$ . If this is not the case, then the behavior of the self-oscillator is somewhat more complicated, but to a certain extent, by employing two-phase output, one can obtain some information as to both the magnitude and the direction of the transient. Under severe situations, a transient at right angles to  $H_0$  can quench the oscillations completely, in which case they will require the normal period of  $(\gamma H_1)^{-1}$  in which to build up.

## VI. Conclusions

Starting from a discussion of optically pumped magnetometers in general, we have concentrated on the two types of self-oscillating rubidium magnetome-

ters represented by the single cell and double cell system. It has been shown that these systems have an intrinsically extremely high signal-to-noise ratio, the possibility of a wide dynamic range, an orientation dependence (with regard to signal-to-noise ratio) which is no worse than other types, and an ability to respond to, and to give data on, extremely rapid field fluctuations. In addition, the double gas cell system has the advantage of a north-south orientation independence, a symmetrized line eliminating possible second-order heading error effects, and the ability to present two-phase information. Magnetometers of both the single and double gas cell types have been flown in space exploration missions and have demonstrated their usefulness. It is expected that these instruments will find increasing use both for outer space and for geophysical explorations.

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## REFEREES

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